2.4 teacher notes

Objectives

- Recognize like terms.
- Use like terms to simplify expressions.
- Add and subtract polynomials both vertically and horizontally.

Vocabulary

Term (Lesson 2-1)
Coefficient (Lesson 2-1)
Algebraic expression (Lesson 2-1)
Variable (Lesson 2-1)
Constant (Lesson 2-1)
Like terms
Trinomial (Lesson 2-1)
Polynomial (Lesson 2-1)
Monomial (Lesson 2-1)
Zero pairs

Prerequisites

Simplifying expressions with rational terms

Simplifying expressions with exponents and roots

Applying algebraic properties

Applying the order of operations

Get Started

- Have students place 3 “x^2” tiles, 4 “x” tiles, and 2 “one” tiles on their desks using the colors that represent positive numbers.
- Ask the students how many of each type of tile they have. 3 “x^2” tiles, 4 “x” tiles and 2 “one” tiles
- Write this as a polynomial. $3x^2 + 4x + 2$
- In order to count the tiles, it was necessary to count each type of tile individually. The tiles that are the same can be called “like tiles”.

Section 1

Expand Their Horizons

In Section 1, students will identify and combine like terms in a polynomial. Combining like terms is necessary for simplifying some algebraic expressions, including adding and subtracting polynomials.

This concept can be related to something as elementary as adding apples. Say the students have green apples, yellow apples, and red apples and want to know how many of each kind they have. Then, tell the students to add the apples of the same color. Although they are all apples, to be "like apples" they must be the same color. By similar reasoning, in order for terms with the same variables to be like terms, the corresponding variables must be raised to the same power.

Common Error Alert

Some students may argue that ab^2 and a^2b are like terms. When these terms are written in expanded form as $a \cdot b \cdot b$ and $a \cdot a \cdot b$, it is evident that the terms are not the same.

In order to give the students a concrete basis for the material, the manipulatives section of this lesson may be shown first. $6x + 3 + 4x + 2$ can be combined using manipulatives. Use 6 green "x" tiles, 3 yellow "one" tiles, 4 green "x" tiles and 2 yellow "one" tiles. Put the "like tiles" together. There will be 10 green "x" tiles and 5 yellow "one" tiles. It is easy for students to see that only the "like tiles" can be combined.

Review the Commutative, Associative, and Distributive properties. Show how these properties are used when expressions are simplified.

In order to combine $6x + 3 + 4x + 2$ algebraically, it is necessary to first use the Commutative Property of Addition and rewrite the problem as $6x + 4x + 3 + 2$. The Associative Property of Addition allows the problem to be grouped as $(6x + 4x) + (3 + 2)$. The result is $10x + 5$.

1 Group the like terms as $(3a + 5a) + (6b - 3b)$ to get $8a + 3b$.

Students at this level are usually still confused by subtraction signs and negative signs. It may be beneficial to rewrite problems such as $6b - 3b$ as $6b + (-3b)$. Note how rewriting subtraction as adding the opposite makes it easier to identify the coefficients. You may also want to suggest to students that they write the understood coefficient of 1 in front of any term without a coefficient.

The same is true when combining like terms. Write $x^2 + x^2$ in expanded form to show that it is not equal to x^4. ($x^2 + x^2 = (x \cdot x) + (x \cdot x) \neq x^4$.) If students still are not convinced, use constants instead of variables. $3^2 + 3^2 = (3 \cdot 3) + (3 \cdot 3) = 9 + 9 = 18$. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$. $18 \neq 81$.

2 All constants are like terms, but if terms have variables, they must have exactly the same variables with exactly the same exponents to be like terms. The like terms are: $3x$ and $-3x$, $2xy$ and $4xy$, and $-y$ and $2y$.

3 Write the terms in expanded form if necessary: $2x^2y^3 = 2 \cdot x \cdot x \cdot y \cdot y \cdot y$, $4x^3y^2 = 4 \cdot x \cdot x \cdot x \cdot y \cdot y$, and $3x^3y^2 = 3 \cdot x \cdot x \cdot x \cdot y \cdot y$. The like terms are $4x^3y^2$ and $3x^3y^2$.

Additional Examples

1. Identify like terms:
$3m^2n^3$, $3m^3n^2$, $6m^3n^2$

The like terms are $3m^3n^2$ and $6m^3n^2$.

2. Simplify:
$8p - 2q + 3p + 4q$

$8p + 3p - 2q + 4q = 8p + 3p + (-2q) + 4q = 11p + 2q$

Section 2

Expand Their Horizons

In Section 2, students will add and subtract polynomials by combining the like terms. Students vary in their preferences for adding polynomials either vertically or horizontally. If they become proficient using both methods, then it may be easier for them later as they begin solving equations.

To find the sum $(x^2 - 6x + 8) + (x^2 + 6x - 8)$, students can use the Commutative and Associative properties. They could rewrite the problem as $(x^2 + x^2) + (-6x + 6x) + (8 - 8)$. This is essentially what they are doing when they identify the like terms and combine them.

To subtract a polynomial, add its opposite. To accomplish this, either of the following interpretations can be used:

- Distribute the minus sign through the second polynomial, or
- Multiply the second polynomial by –1.

By either interpretation, the sign of each term in the second polynomial is changed. $(x^2 - 6x + 8) - (x^2 + 6x - 8)$ becomes $(x^2 - 6x + 8) + (-x^2 - 6x + 8)$. When the like terms are combined, the result is $-12x + 16$.

Common Error Alert

Students may fail to distribute the minus sign through the entire polynomial when subtracting polynomials. They may change the sign of the first term and leave the other terms in their original form. Stress to students that they are subtracting the entire polynomial.

Polynomial subtraction can be done in the vertical format. The following example (not part of the video) shows subtraction in the vertical format.

Simplify: $(x^2 - 3) - (x^2 + 3x - 2)$

$$\begin{array}{r} x^2 \qquad\quad - 3 \\ \underline{-\,(x^2 + 3x - 2)} \end{array} \qquad \begin{array}{r} x^2 \qquad\quad - 3 \\ \underline{+\;(-x^2 - 3x + 2)} \\ -3x - 1 \end{array}$$

4 If students are struggling, use manipulatives for this problem. The first trinomial is represented by one blue "x^2" tile, 5 red "x" tiles, and 6 yellow "one" tiles. The second polynomial is represented by one blue "x^2" tile, 5 red "x" tiles, and 6 red "one" tiles. To subtract, change the color of each of the tiles representing the second polynomial. Group like tiles together. A zero pair consists of one positive and one negative "like tile". Remove all zero pairs and count the tiles that are left. There should be 12 yellow "one" tiles.

Look Beyond

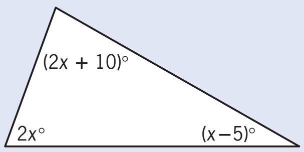
In future lessons, students will be combining like terms to solve equations. In equations such as $3x + 4x = 5 + 9$, the like terms must be combined before the equation can be solved. This equation becomes $7x = 14$.

In a future algebra course, students will also be dividing polynomials. Just as subtraction is a necessary step in dividing numbers, subtracting polynomials is a necessary step in dividing polynomials.

Connections

Combining terms to solve equations is needed in most mathematics courses. To find the angle measures for the triangle shown below, we use the fact that the sum of the angle measures in a triangle is 180°.

$$\begin{aligned}
2x + (2x + 10) + (x - 5) &= 180 \\
2x + 2x + x + 10 - 5 &= 180 \\
5x \qquad\quad + 5 &= 180 \\
5x \qquad\qquad\quad &= 175 \\
x \qquad\qquad\quad &= 35
\end{aligned}$$



$x = 35$. $2x = 70$, $2x + 10 = 80$, and $x - 5 = 30$. The angle measures are 70°, 80°, and 30°.

Additional Examples

1. Add:
$4n^2 + 2n - 3$ and $5n^2 - 3n + 2$

Horizontally:
$(4n^2 + 2n - 3) + (5n^2 - 3n + 2)$
$4n^2 + 5n^2 + 2n - 3n - 3 + 2$
$9n^2 - 1n - 1$
$9n^2 - n - 1$

Vertically:

$$\begin{array}{r}
4n^2 + 2n - 3 \\
+\ 5n^2 - 3n + 2 \\
\hline
9n^2 - n - 1
\end{array}$$

2. Subtract:
$(5x^2 - 6) - (3x^2 + 2x - 9)$

Horizontally:
$(5x^2 - 6) - (3x^2 + 2x - 9)$
$5x^2 - 6 + (-3x^2 - 2x + 9)$
$5x^2 - 3x^2 - 2x - 6 + 9$
$2x^2 - 2x + 3$

Vertically:

$$\begin{array}{r}
5x^2 + 0x - 6 \\
-\ (3x^2 + 2x - 9) \\
\hline
\end{array}
\qquad
\begin{array}{r}
5x^2 + 0x - 6 \\
+\ (-3x^2 - 2x + 9) \\
\hline
2x^2 - 2x + 3
\end{array}$$