$$\Delta = .00\,\pi + \frac{1}{200000} \cdot \frac{1}{\sqrt{xy}}$$

Objectives

- **Recognize and use the Commutative and Associative Properties of Addition and Multiplication.**

- **Recognize the identity elements and inverses for addition and multiplication and use their respective properties.**

- **Recognize and use the Distributive Property of Multiplication over Addition.**

$$\Omega\,\frac{1}{15750} \qquad \overline{5-6} \mid \sqrt{xy}\,\frac{1}{12}\,\Delta$$

2.3 teacher notes

Vocabulary

Commutative Property of Addition
Associative Property of Addition
Commutative Property of Multiplication
Associative Property of Multiplication
Identity Property of Addition
Zero Property of Multiplication
Identity Property of Multiplication
Multiplicative Inverse Property
Reciprocal
Additive Inverse Property
Distributive Property

Prerequisites

Simplifying expressions using the order of operations

Simplifying operations with rational numbers

Get Started

- Fill in the blank: $2 + 3 = 3 +$ _____. **2**

- Fill in the blank: $3 +$ _____ $= 3$. **0**

- Fill in the blank: $4 +$ _____ $= 4$. **0**

- Today you will learn the names of the algebraic properties that allow you to work problems such as these.

Section 1

Expand Their Horizons

Algebraic properties are a set of rules used to rewrite expressions and can be used when solving equations and inequalities. One way to stress the importance of learning these properties is to allow students to simplify expressions without knowing the properties. In this section, students will learn the Commutative and Associative Properties of Addition and Multiplication.

Using simple numbers or manipulatives to provide concrete examples of these properties can be helpful. A manipulatives lesson using algebra tiles is provided. Beginning the lesson with this section and completing the practice portion before moving on to the core lesson may be helpful to students. You can provide other numerical examples like the ones in the Getting Started Section to demonstrate other properties instead of using algebra tiles.

Use manipulatives to illustrate the Commutative Property of Addition, $3 + 4 = 4 + 3$. On one side of the mat, place 3 of the "one" tiles first and then 4 of the "one" tiles. On the other side of the mat, place the group of 4 tiles first and then the group of 3 tiles.

The Commutative and Associative Properties could be used to solve a problem such as $(0.2)(0.6)(5)$ with mental calculations. Rewrite the problem as $(0.2)(5)(0.6)$ using the Commutative and Associative Properties of Multiplication, $(0.2)(5) = 1$. So $(0.2)(5)(0.6) = (1)(0.6)$ and $(1)(0.6) = 0.6$ by the Multiplicative Identity Property.

Common Error Alert

Students often confuse the Commutative and Associative Properties. The Commutative Property deals with order. The Associative Property deals with grouping.

1 ▷ The order of the numbers is the same on both sides of the equation. The parentheses, however, group different numbers. This is the Associative Property of Addition.

2 ▷ The parentheses are around the same numbers, 3 and 5, on both sides of the equation. However, the right side of the equation is $3 \cdot 5$ and the left side of the equation has $5 \cdot 3$. This is the Commutative Property of Multiplication.

Many students assume that the Commutative and Associative Properties apply to all operations. If necessary, have students use a calculator to correct this error. Have them use a calculator and key in $5 - 7$ and $7 - 5$ and then compare the answers. Because $2 \neq -2$, you can show students that subtraction is not commutative. Likewise, the problems $12 \div 2$ and $2 \div 12$ will also show that division is not commutative. Use this to show that the order in which problems are calculated is important.

Additional Examples

1. **Complete the equation to illustrate the Commutative Property of Multiplication.**
   $5(0.19) =$

   The Commutative Property changes the order so, $5(0.19) = \underline{0.19(5)}$

2. **Which property is illustrated by this equation?**
   $(4 + 3) + 6 = (3 + 4) + 6$

   Although grouping symbols are used, they remain around the same numbers. However, the order of the 3 and 4 is changed. Hence, this illustrates the Commutative Property of Addition.

Section 2

Expand Their Horizons

In Section 2, students will learn the Identity and Inverse Properties for Addition and Multiplication.

Students may have trouble distinguishing between the Identity Property and the Inverse Property. If someone asks you for I.D., they want to know your identity. They want to know who you are. The same is true in mathematics. Adding zero or multiplying by one does not change the value of any number.

The word "inverse" can be related to the word "invert". When something is inverted, it is turned upside down. The same is true when finding the multiplicative inverse of a number; the multiplicative inverse of any nonzero number is its reciprocal. For addition, "inverse" can be related to "reverse" or "opposite". The additive inverse of any number is its opposite.

The product of a non-zero number and its multiplicative inverse equals the multiplicative identity, 1. The sum of a number and its additive inverse equals the additive identity, 0.

3 $7 + 0 = 7$. In this addition problem, the 7 retains its identity after being added to 0. This is the Identity Property of Addition.

4 The Zero Property of Multiplication states that if 0 is multiplied by any number, the result is 0. Any number will work as an example. For instance, $3(0) = 0$, $0.07(0) = 0$.

Common Error Alert

Students may want to change the sign of the number to find the multiplicative inverse. A number times its multiplicative inverse is equal to 1, a positive number. Therefore, a number and its multiplicative inverse must be the same sign.

Additional Examples

1. **Fill in the blank with the correct number. The multiplicative inverse of –3 is _____.**

   The multiplicative inverse of –3 is $-\frac{1}{3}$.

2. **Write an equation that illustrates the Identity Property of Multiplication.**

   Possible answer: $4 \cdot 1 = 4$

Section 3

Expand Their Horizons

In Section 3, students learn to use the Distributive Property of Multiplication Over Addition.

The Distributive Property could be used to solve a problem such as $32 \cdot 101$ mentally by simplifying the expression $32 (100 + 1)$. This distributes to $32 \cdot 100 + 32 \cdot 1$. $3200 + 32 = 3232$.

The problem $32 \cdot 99$ could be solved in the same manner by distributing 32 in the expression $32(100 - 1) = 3200 - 32 = 3168$.

5 Have students find the solution to this equation to show the usefulness of the Distributive Property. $12(5 + 9) = 12(5) + 12(9) = 60 + 108 = 168$ or $12(5 + 9) = 12(14) = 168$.

Connections

When a person is bidding for items at an auction, he or she must be able to quickly compute the cost of the item. For example, if the person is bidding on 510 pounds of apples, at $0.40 a pound, they must be able to mentally calculate the value of the apples. The Distributive Property of Multiplication over Addition makes this process much easier. $0.40(500 + 10) = 200 + 4 = \$204$.

Look Beyond

In later lessons students will apply these properties using variables. For example, they will use the Distributive Property of Multiplication over Addition to simplify an expression such as $4(a + 5)$ into $4a + 20$.

Additional Examples

1. **Complete the equation so that the result illustrates the Distributive Property of Multiplication over Addition.**
   $4(12 + 9) =$

   $4(12 + 9) = 4(12) + 4(9)$
   $ = 48 + 36$
   $ = 84$

2. **Complete the equation so that the result illustrates the Distributive Property of Multiplication over Addition.**
   $3(14 + 9) =$

   $3(14 + 9) = 3(14) + 3(9)$
   $ = 42 + 27$
   $ = 69$