18.3 teacher notes

Objective

- Solve application problems in which a radical equation must be solved.

Vocabulary

Pythagorean Theorem

Hypotenuse

Prerequisites

Solving multi-step radical equations

Solving linear equations

Rounding decimal numbers

Get Started

- Divide students into pairs. Ask student pairs to list the operations, in order, that would be performed to evaluate the expression given a value for the variable x: $2\sqrt{3x + 1} - 7$. Some students may find it helpful to substitute a number for the variable to make this determination.
- After all pairs have completed their list, ask pairs to volunteer their list of steps. Write the correct list of steps on the board: multiply by three, add one, take the square root of the quantity, multiply the quantity by two, and subtract seven.
- Ask the same student pairs to solve the equation $2\sqrt{3x + 1} - 7 = 1$.
- After several pairs arrive at the correct solution, ask one pair to demonstrate the steps. Cross off the previous list of steps, in reverse

order, as they correspond to their inverse operations in the equation's solution. Correct solution should be as follows:

$2\sqrt{3x + 1} = 8$	Add 7.
$\sqrt{3x + 1} = 4$	Divide the quantity by 2.
$3x + 1 = 16$	Square both sides (on the left, squaring eliminates the radical).
$3x = 15$	Subtract 1.
$x = 5$	Divide by 3.

- Notice the operations used to solve the equation are in the reverse order of the operation that would be used to evaluate the related expression.

Section 1

Expand Their Horizons

In Section 1, students will apply techniques for solving multi-step radical equations to applications. To solve the equation, students should:

1. Isolate the radical on one side of the equation using inverse operations.
2. Eliminate the radical by raising each side of the equation to the appropriate power.
3. If necessary, solve the resulting equation.

The appropriate power, to eliminate a square root radical, is two; for a cube root radical, the power is three; for a fourth root radical, the power is four; and so on.

The first solved equation in the lesson is $s = 5.5\sqrt{0.75m}$, a formula for determining the speed of a car, s, in miles per hour, given the length of the skid mark, m, in feet.

Consider a speed of 55 mph. As with any formula, the first step is to substitute given values into the equation: $55 = 5.5\sqrt{0.75m}$. Then, the variable must be isolated in the radical expression. The radical sign has no index indicated. Remind students this indicates an index of two which is understood. To undo this square root operation, raise both sides of the equation to the second power. Then, solve the equivalent linear equation. The solution is rounded to the nearest whole number. Thus, at 55 miles per hour, when the brakes are applied, there is a skid mark of about 133 feet.

Remind students solutions to radical equations must be checked in the original equation; a true statement indicates a correct answer. Because the original solution was an estimate, use the "approximately equals" symbol to check the answer.

$$55 \approx 5.5\sqrt{0.75(133)}$$
$$55 \approx 5.5\sqrt{99.75}$$
$$55 \approx 5.5(9.99)$$
$$55 \approx 54.95$$

Common Error Alert

Students may not feel comfortable using approximations in checking an answer. Tell students their answers should be within one-tenth of the number they are checking against. For example, 54.95 is within one-tenth of 55. Students should check their work if their answer contains a greater error than this.

Notice this answer does not check exactly, but it is approximately correct. Due to rounding within the solution and for the final result, the check will not be exact but should be close enough to determine the solution was correct, as it is in this equation. The exact solution to this problem was $133\frac{1}{3}$, which yields $55 = 55$ when checking. However, for application problems, decimal answers and rounding are generally preferred.

In application problems, determine if the obtained answer is reasonable. For example, if a car, at a given speed, produced a skid mark which was a negative number, this solution would be invalid because length cannot be negative. Such a result could indicate either a mistake was made in solving the equation, or the equation had no solution.

Connections

Bankers and financial advisors can use equations with radicals to make investment decisions. Architects can use radical equations to form various shapes for interesting window or doorway designs. The Additional Problems section gives examples of these connections to radicals in the real world.

1 The same skid mark formula is used for a car traveling at 60 miles per hour. The square of 10.91 is approximately 119.03, and squaring a square root radical cancels the operation, so the equation becomes $119.03 \approx 0.75m$. To solve this linear equation, divide both sides by the coefficient to get the solution $159 \approx m$. Thus, at 60 mph, a car would leave about a 159-foot skid mark. This scenario may be expanded by asking students to determine whether the result seems reasonable by comparing the speeds of the cars and the lengths of the skid marks in the two scenarios. The length of the skid mark is directly proportional to the square of the speed. The related and equivalent equation for the original formula is $m = \frac{16}{363}s^2$.

2 Substituting 45 for the distance gives the equation $45 = 1.17\sqrt{h}$. To isolate the radical, divide both sides by 1.17 and round the result to get $38.46 \approx \sqrt{h}$. Squaring both sides removes the radical on the right and yields $1479 \approx h$. Thus, the height of the summit is approximately 1,479 feet when the distance to the horizon is 45 miles.

3 To find the temperature at which sound travels 400 meters per second, substitute 400 for v. Isolate the radical; divide both sides by 20 to get $20 = \sqrt{t + 273}$. To remove the square root radical, square both sides. Because 20^2 equals 400, the equation becomes $400 = t + 273$. It is only a coincidence the number on the left became 400 again! Solving the linear equation that remains, 273 is subtracted from both sides to produce $127 = t$. Thus, the air temperature is 127°C if sound travels at 400 m/s.

4 Using the Pythagorean Theorem, $c = \sqrt{a^2 + b^2}$, substitute 13 for c, the hypotenuse, and five for a, the leg. Both a and b represent legs; thus, five could be substituted for b instead. The value 5^2 equals 25; this gives the equivalent equation $13 = \sqrt{25 + b^2}$. Because the radical is already isolated, square both sides to eliminate the square root radical: $169 = 25 + b^2$. There are two solutions: $12 = b$ and $-12 = b$, but b represents a distance, so it cannot be a negative number. Therefore, the ladder reaches 12 feet up the side of the building.

Look Beyond

Many branches of mathematics utilize equations with radicals. Trigonometry makes great use of radicals since the study is based on triangles, and many of the formulas are derived from the Pythagorean Theorem, which was demonstrated in this lesson. Formulas from the Law of Cosines, $c = \sqrt{a^2 + b^2 - 2ab\cos C}$; to trigonometric identities, $\sin\theta = \pm\sqrt{1 - \cos^2\theta}$; to half-angle trigonometric functions, $\cos\frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$ are used in trigonometry to solve triangles and equations. The techniques utilized in this section are used to solve those equations.

Additional Examples

1. The formula for the area of a quadrilateral inscribed in a circle is given below, where *a*, *b*, *c*, and *d* are the sides of the quadrilateral and *Q* is the area. Find the length of the missing side if the area is 28.3 sq cm, two sides have length 8.9 cm, and a third side has length 1.4 cm.

$Q = \sqrt{abcd}$

$28.3 = \sqrt{(8.9)(8.9)(1.4)d}$	Substitute known values.
$28.3 \approx \sqrt{110.9d}$	Simplify the multiplication under the radical.
$28.3^2 \approx \sqrt{110.9d}^{\,2}$	Square both sides.
$800.9 \approx 110.9d$	Simplify.
$d \approx 7.2$	Divide both sides by the coefficient.

Therefore, the length of the missing side is about 7.2 cm.

2. The given formula is used to determine what interest rate, *r* in decimal, is necessary to turn a principal investment, *P*, into the amount, *A*, if that interest is compounded annually for a time in years, *t*. Determine what the resulting amount would be from an initial investment of $500 compounded at 4% annually after 30 years.

$r = \sqrt[t]{\frac{A}{P}} - 1$

The rate, 4%, in decimal form is 0.04; substitute this into the equation for r. The time is 30 years; substitute this into the equation for t. The principal is $500; substitute this into the equation for P.

$0.04 = \sqrt[30]{\frac{A}{500}} - 1$	
$1.04 = \sqrt[30]{\frac{A}{500}}$	Add 1 to both sides.
$3.24 \approx \frac{A}{500}$	Raise each side to the 30th power.
$1620 \approx A$	Multiply both sides by 500.

An investment of $500 at 4% compounded annually for 30 years will amount to $1620.