18.2 teacher notes

Objectives

- Solve multi-step radical equations.
- Determine whether a radical equation has a solution by checking.

Vocabulary

Radical equation (Lesson 18-1)
Isolate (Lesson 3-1)
Principal square root (Lesson 1-4 see square root)
Radical sign (Lesson 1-4)

Prerequisites

Solving equations using inverse operations

Squaring, cubing, and raising numbers to the fourth power

Solving one-step radical equations

Get Started

- Write the equation $3\sqrt{x} + 1 = 7$ on the board. Then, tape a piece of construction paper or an index card over the radical.
- To begin, ask students to solve the equation on their own paper by drawing a rectangle to represent the paper rectangle. Tell them to solve for the variable (the rectangle).
- Ask students to describe each step in the solution. Write their steps on the board, using additional paper rectangles to replace the radicals. The steps in the solution should include $3\square + 1 = 7$; $3\square = 6$; $\square = 2$.
- Once the solution has been found, remove the paper to reveal $\sqrt{x}$. Then, remove each subsequent paper and write $\sqrt{x}$ in its place. The equation is now $\sqrt{x} = 2$.
- Ask the class to continue solving the equation. (The solution is $x = 4$). Students should check their answer in the original equation $3\sqrt{x} + 1 = 7$.
- Say, "In this lesson, we will solve radical equations for which more than one step is necessary to solve. In these equations, we will isolate the radical, eliminate it, and then, solve the resulting equation."

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Section 1

Expand Their Horizons

In this lesson, students will solve multi-step radical equations. Students will encounter equations like $\sqrt{b} - 5 = -7$, in which the radical is subject to subtraction (addition) and $\sqrt[3]{5x + 6}$, in which the radicand shows the variable subject to multiplication and then, to addition (subtraction).

Solving each equation in this lesson uses the same fundamental idea: isolate the radical on one side of the equation using inverse operations, eliminate the radical by raising each side of the equation to the appropriate power, and then, if necessary, solve the resulting linear equation.

For each problem in Section I, one operation is performed on the radical. All problems involve a square root. The radical is either added to, subtracted from, multiplied by a number, or divided by a number. Each equation can be solved in two steps. To solve each equation, use inverse operations to isolate the radical and then, square both sides.

Visual learners may find it helpful to carry the "Get Started" activity to the lesson problems. The use of the radical symbol in a multi-step equation can be overwhelming for some students. After copying the problems onto their papers, the students should replace the entire radical expression with a rectangle or other symbol. Then, they should solve for the rectangle, replacing the symbol with the radical only when the equation shows the symbol equal to a real number. They can then continue by solving the resulting one-step radical equation.

Like one-step radical equations, multi-step radical equations must always be checked. Squaring both sides of an equation can result in extraneous solutions; such solutions must be eliminated. Remind students they must check their answers in the original equation (the given equation). Checking their answers in subsequent steps might lead them to preserve an extraneous solution.

In the first lesson example, $\sqrt{b} - 5 = -7$, five is added to each side of the equation to isolate the radical. The resulting equation is $\sqrt{b} = -2$. Point out to students they should recognize by inspection the equation has no solution, because the square root of a number is always positive.

1 In this equation, one is added to the radical. Isolate the radical by subtracting one from both sides of the equation. The resulting equation is $\sqrt{x} = 3$. Square both sides of the equation to get $x = 9$. Check the solution in the equation $\sqrt{9} + 1 \stackrel{?}{=} 4$. Because $3 + 1 = 4$, the solution checks.

2 This equation shows the radical divided by six. Eliminate the division by multiplying both sides by six to get $\sqrt{m} = 12$. At this point, the radical is isolated on the left side of the equation, and both sides should be squared to solve. The solution is $m = 144$. Check the solution in the equation $\frac{\sqrt{144}}{6} \stackrel{?}{=} 2$.

Because $\frac{12}{6} = 2$, the solution checks. Students may mention other methods for solving the problem, such as cross-multiplying or multiplying by the LCM of the denominators. All methods use the same operation: multiplying both sides of the equation by six.

Common Error Alert

Students may fail to isolate the radical before squaring to eliminate the radical. For the equation $\sqrt{x} + 1 = 4$, they may incorrectly eliminate the radical by writing $x + 1 = 16$. If this becomes a consistent problem for some students, suggest they use the technique of replacing the radical with another symbol until the equation shows the symbol equal to a number.

Additional Examples

1. $\frac{2}{5}\sqrt{x} = 4$

The radical is multiplied by a fraction. To eliminate the fraction, multiply both sides of the equation by its reciprocal.

Solve:

$$\frac{2}{5}\sqrt{x} = 4$$
$$\sqrt{x} = \frac{5}{2} \cdot 4$$
$$\sqrt{x} = 10$$
$$(\sqrt{x})^2 = (10)^2$$
$$x = 100$$

Check:

$$\frac{2}{5}\sqrt{100} \stackrel{?}{=} 4$$
$$\frac{2}{5} \cdot 10 \stackrel{?}{=} 4$$
$$4 = 4$$

2. $-7\sqrt{x} = 14$

Divide both sides of the equation by –7 to isolate the radical. The resulting equation is $\sqrt{x} = -2$. If students recognize at this point that the equation has no solution, no further steps are needed.

Solve:

$$-7\sqrt{x} = 14$$
$$\sqrt{x} = \frac{14}{-7}$$
$$\sqrt{x} = -2$$
$$(\sqrt{x})^2 = (-2)^2$$
$$x = 4$$

Check:

$$-7\sqrt{4} \stackrel{?}{=} 14$$
$$-7 \cdot 2 \stackrel{?}{=} 14$$
$$-14 \neq 14$$

The equation has no solution.

Section 2

Expand Their Horizons

In Section 2, multi-step radical equations are explored further. In the problems for this section, cube roots appear; more than one operation is performed on the radical, or the radicand is an expression involving one or more operations. Each of the equations in this section requires more than two steps to solve. However, the basic strategy is the same: isolate the radical and then, eliminate it. After eliminating the radical, solve the resulting linear equation to find the solution.

3 This equation shows a cube root radical that is already isolated on the left side of the equation. To eliminate the radical, cube both sides of the equation to get $5x + 6 = 1$. Solve the resulting equation by subtracting six from both sides ($5x = -5$) and then, by dividing both sides by five. The solution is $x = -1$. Check the solution in the equation $\sqrt[3]{5(-1) + 6} \stackrel{?}{=} 1$. Simplify the radicand to get $\sqrt[3]{-5 + 6} \stackrel{?}{=} 1$; $\sqrt[3]{1} \stackrel{?}{=} 1$. Because the cube root of one is one, the solution checks.

4 Isolate the radical by subtracting 10 from both sides of the equation ($\sqrt{5p} = 15$). Eliminate the radical by squaring both sides of the equation ($5p = 225$). Finally, solve the resulting equation by dividing both sides by five ($p = 45$). Check the solution. $\sqrt{5 \cdot 45} + 10 \stackrel{?}{=} 25$; $\sqrt{225} + 10 \stackrel{?}{=} 25$; $15 + 10 \stackrel{?}{=} 25$. The solution checks.

Connections

Many real-life formulas take the form of radical equations. In the next lesson, students will see how radical equations can be solved to determine quantities like length of skid marks left at an accident scene, distance to the horizon, length of a pendulum's swing, and more.

Look Beyond

The study of radical equations is essential to developing and using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. The distance formula is an integral part of the study of analytic geometry, in which geometric figures are studied on the coordinate plane.

Additional Examples

1. $\sqrt{1 - 6x} = 7$

Since the radical is isolated on the left side of the equation, square both sides of the equation to eliminate it. Then, solve the resulting equation.

Solve:

$$\sqrt{1 - 6x} = 7$$
$$(\sqrt{1 - 6x})^2 = 7^2$$
$$1 - 6x = 49$$
$$-6x = 48$$
$$x = -8$$

Check:

$$\sqrt{1 - 6(-8)} \stackrel{?}{=} 7$$
$$\sqrt{1 + 48} \stackrel{?}{=} 7$$
$$\sqrt{49} \stackrel{?}{=} 7$$
$$7 = 7$$

2. $2\sqrt{3x} - 4 = 8$

First, isolate the radical. The isolation requires two steps: subtracting four, then dividing by two. Once the radical is isolated; square both sides to eliminate it. Solve the resulting equation ($3x = 36$) to find the solution.

Solve:

$$2\sqrt{3x} - 4 = 8$$
$$2\sqrt{3x} = 12$$
$$\sqrt{3x} = 6$$
$$(\sqrt{3x})^2 = (6)^2$$
$$3x = 36$$
$$x = 12$$

Check:

$$2\sqrt{3 \cdot 12} - 4 \stackrel{?}{=} 8$$
$$2\sqrt{36} - 4 \stackrel{?}{=} 8$$
$$2 \cdot 6 - 4 \stackrel{?}{=} 8$$
$$12 - 4 \stackrel{?}{=} 8$$
$$8 = 8$$