18.1 teacher notes

Objectives

- Solve one-step radical equations.
- Determine whether a solution is extraneous by checking.

Vocabulary

Radical equation
Radical sign (Lesson 1-4)
Radicand (Lesson 1-4)
Extraneous solution

Prerequisites

Solving equations using inverse operations

Squaring, cubing, and raising numbers to the fourth power

Get Started

- Ask students to consider the equations $x + 3 = 5$ and $-4x = 8$.
- Ask, "How would you solve each equation?" Students' response should include the term inverse operations.
- Remind students the goal in solving equations is to "undo" operations on x by using inverse operations. Point out in solving $x + 3 = 5$, they "undo" adding three by subtracting three, and in solving $-4x = 8$, they "undo" multiplying by -4 by dividing by -4. In each equation, performing inverse operations leaves an equation of the form x equals some number; this is the solution.
- Also, remind students the rules of equality require that when they operate on one side of an equation, the same operation must be performed on the other side of the equation.

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- Next, write the equation $\sqrt{x} = 3$ on the board. Ask, "In this equation, what operation needs to be undone?" The operation "taking the square root of x" needs to be undone. Ask students for the inverse operation. Squaring is the inverse operation of taking the square root.
- Say, "In this lesson, we will solve equations in which the operation on the variable involves taking the square, cube, or fourth root. In each equation, we will use an inverse operation to "undo" that operation.

Section 1

Expand Their Horizons

In this lesson, students solve equations in which the variable appears under the radical. Each equation can be solved in one step. To solve each equation, the radical operation is eliminated by raising both sides of the equation to the appropriate power. When the variable is under a square root radical, both sides are squared. When the variable is under a cube root radical, both sides are cubed. The result of the operation is an equation with the variable alone on one side and a constant on the other side. This equation shows the solution.

Common Error Alert

When both sides of an equation are raised to a power, extraneous solutions are sometimes created. That is, perfectly legitimate operations on both sides of the equation can sometimes lead to extra solutions. After solving a radical equation, it is essential to check answers. For example, consider the equation $\sqrt{x} = 3$. Square both sides: $x = 9$. Now square both sides again: $x^2 = 81$. The solution set to this equation is $x = -9$ or $x = 9$. The variable x, however, cannot be -9 in the original equation because there is no real number that is the square root of a negative number. The value -9 is an extraneous solution.

Students are likely to solve the equations in Section I by inspection. When solving an equation like $\sqrt{x} = 3$, students may think to themselves "the square root of what number is three?" Solving by inspection is a valid technique but be sure to emphasize the use of inverse operations as a more systematic reliable method of solving equations.

Before beginning the lesson, it may be helpful to review perfect square numbers with the class. Make and display a chart showing the squares of whole numbers ($1^2 = 1$, $2^2 = 4$, etc.). Consider asking students to memorize the perfect squares of the whole numbers from zero to 20. Students who have these numbers memorized can concentrate better on the task at hand—solving radical equations using inverse operations.

1 To solve, use an inverse operation to undo the "square root" operation. In this case the inverse operation is to square both sides of the equation. The square of the square root of x is x, and the square of 12 is 144. The solution is $x = 144$. Check this by substituting the value back into the original equation. The value $\sqrt{144}$ equals 12.

2 To solve, eliminate the radical by squaring both sides of the equation. The resulting equation is $x = 49$, which shows the solution. Check by substituting the solution into the original equation. The value $\sqrt{49}$ equals seven.

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Additional Examples

1. $\sqrt{x} = 5$

Solve:

$$\sqrt{x} = 5$$
$$(\sqrt{x})^2 = 5^2$$
$$x = 25$$

Check:

$$\sqrt{25} \stackrel{?}{=} 5$$
$$5 = 5$$

2. $\sqrt{x} = 8$

Solve:

$$\sqrt{x} = 8$$
$$(\sqrt{x})^2 = 8^2$$
$$x = 64$$

Check:

$$\sqrt{64} \stackrel{?}{=} 8$$
$$8 = 8$$

Section 2

Expand Their Horizons

In Section 2, radical equations containing negative signs are solved. Students will investigate problems like $\sqrt{x} = -4$ and $-\sqrt{x} = -9$. Solve the equations in this section like those in Section I. Eliminate the radical by squaring both sides of the equation. Parentheses are necessary when squaring an expression like $-\sqrt{x}$ or when squaring a negative number.

Several of the problems in this section take the form $\sqrt{x} = a$, where a is a negative number. Since the square root radical symbol indicates the principal (nonnegative) square root, it cannot equal a negative number. Therefore, such an equation never has a solution. Encourage students to make this generalization after solving several problems of this type.

Contrast the form $\sqrt{x} = a$ $(a < 0)$ with the form $-\sqrt{x} = a$ $(a < 0)$, which *does* have a solution. It may be helpful for students to read negative signs in this section as "the opposite of." Read $-\sqrt{x} = -9$ as "the opposite of the square root of x is equal to the opposite of nine." If the opposite of the square root of x is the same as the opposite of nine, then the (principal) square root of x must be nine. So, 81 is the solution.

3 To solve, square both sides to eliminate the radical. The result is $x = 36$. When the solution is checked, the equation is $\sqrt{36} \stackrel{?}{=} -6$. Because the radical sign indicates the principle square root, $\sqrt{36} = 6$, and the solution does not check. The equation has no solution. Students, who have generalized the equation $\sqrt{x} = a$ has no solution for values of a less than zero, can apply the idea here.

Common Error Alert

Students may fail to check their answer by substituting these values into the original equation. Remind them it is imperative to check their solutions to rule out extraneous solutions.

4 Square both sides of the equation. Parentheses are necessary since the expressions on each side contain negative signs. The result is $x = 121$. To check, substitute 121 for x to confirm the equation $-\sqrt{121} = -11$ is true. Simplify the radical on the left side of the equation to arrive at $-11 = -11$. The answer is correct.

Common Error Alert

Students may fail to use parentheses when squaring both sides of the equation or may not see how the square of $-\sqrt{x}$ is x. Show them because $-\sqrt{x}$ represents a negative number (or zero), its square is positive (or zero).

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Additional Examples

1. $\sqrt{x} = -2$

Solve:

$$\sqrt{x} = -2$$
$$(\sqrt{x})^2 = (-2)^2$$
$$x = 4$$

Check:

$$\sqrt{4} \stackrel{?}{=} -2$$
$$2 \neq -2$$

The equation has no solution.

2. $-\sqrt{x} = 6$

Solve:

$$-\sqrt{x} = 6$$
$$(-\sqrt{x})^2 = 6^2$$
$$x = 36$$

Check:

$$-\sqrt{36} \stackrel{?}{=} 6$$
$$-6 \neq 6$$

The equation has no solution.

Section 3

Expand Their Horizons

In Section 3, the study of one-step radical equations is expanded to include cube roots and fourth roots. Cube root and fourth root equations are solved in the same manner as square root equations. Inverse operations are used to undo the operation indicated by the radical. The inverse of taking the cube root is cubing; the inverse of taking the fourth root is to raise to the fourth power.

Take a few moments to review the cube root and fourth root notation. The index is the number "tucked into" the nook of the radical and indicates which root is to be taken. When a square root is indicated, no index is shown. An index of three indicates the cube root, and an index of four indicates the fourth root.

Review the cubes and fourth powers of whole numbers. If a chart was made for perfect squares, add a column for cubes and fourth powers. Write $1^3 = 1$, $2^3 = 8$, etc. and $1^4 = 1$, $2^4 = 16$, etc. It will be helpful if students can quickly identify cubic and fourth power numbers.

In Section 2, some students may have generalized that $\sqrt{x} = a$ $(a < 0)$ has no solution. Point out to those students $\sqrt[3]{x} = a$ $(a < 0)$ does have a solution. The cube root of a negative number is negative; the cube root of a positive number is positive; and the cube root of zero is zero. So, $\sqrt[3]{x} = a$ always has a solution, regardless the sign of a. Like square roots, $\sqrt[4]{x} = a$ $(a < 0)$ does not have a solution. The fourth root of a number is always positive.

5 To solve, raise each side to the fourth power. On the left side, the inverse operation undoes the fourth root operation, and the result is x. On the right side, 5^4 is $5 \cdot 5 \cdot 5 \cdot 5$ or 625. The solution is $x = 625$. To check, substitute 625 for x in the original equation to get $\sqrt[4]{625} = 5$, or $5 = 5$. The solution checks.

Common Error Alert

When students raise each side of the equation to the fourth power, they often multiply the non-radical side of the equation by four. Remind them a number to the fourth power indicates the same factor should be multiplied together four times.

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Additional Examples

1. $\sqrt[3]{x} = -3$

Solve:

$$\sqrt[3]{x} = -3$$
$$(\sqrt[3]{x})^3 = (-3)^3$$
$$x = -27$$

Check:

$$\sqrt[3]{-27} \stackrel{?}{=} -3$$
$$-3 = -3$$

2. $-\sqrt[4]{x} = -1$

Solve:

$$\sqrt[4]{x} = -1$$
$$(\sqrt[4]{x})^4 = (-1)^4$$
$$x = 1$$

Check:

$$\sqrt[4]{1} \stackrel{?}{=} -1$$
$$1 \neq -1$$

The equation has no solution.

Look Beyond

This lesson shows another class of operations that can be "done" to a variable and "undone" using inverse operations. Students have had experience using inverse operations of addition, subtraction, multiplication, and division in previous lessons, and here, they extend the idea to powers. In future courses, the idea will be extended by using inverse operations to undo operations such as trigonometric, logarithmic, and exponential operations. Students will also see how equations can be written without radicals using fractional exponents. For example, the expression $\sqrt{x}$ can be written as $x^{\frac{1}{2}}$. The expression $\sqrt[3]{x}$ can be written as $x^{\frac{1}{3}}$. To solve the equation $\sqrt[3]{x} = 2$, rewrite the radical expression as $x^{\frac{1}{3}} = 2$. To solve, the goal is to achieve a power of one on x; so, cube both sides. Use the laws of exponents on the left side of the equation.

$$x^{\frac{1}{3}} = 2$$
$$\left(x^{\frac{1}{3}}\right)^3 = 2^3$$
$$x^{\frac{1}{3} \cdot 3} = 8$$
$$x^1 = 8$$
$$x = 8$$

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