17.3 teacher notes

Objective

- Multiply radicals.

Prerequisites

Adding and subtracting radicals

Combining like terms

Using the Distributive Property of Multiplication over Addition

Using the FOIL Method

Simplifying radicals using the Product Property of Square (Cube) Roots

Vocabulary

Product Property of Square Roots (Lesson 17-1)
Product Property of Cube Roots (Lesson 17-1)
Cube root (Lesson 17-1)
Index (Lesson 17-1)
Perfect square (Lesson 17-1)
Distributive Property of Multiplication over Addition (Lesson 2-3)
Perfect cube (Lesson 17-1)
FOIL Method (Lesson 11-5)

Get Started

- Break the class into three groups. Each group will simplify a different problem.
- Give the first group the expression $7x^3 \cdot 5x^2$; give the second group the expression $5x(x - 4)$; and give the third group the expression $(x + 3)(x + 6)$.
- After all three groups finish, ask a spokesperson from each to demonstrate the process used to simplify each expression.
- The first group's expression equals $35x^5$. The student should explain how to use the Exponent Rule to simplify this expression. Point out that the coefficients and variables are multiplied separately.
- The second group's expression equals $5x^2 - 20x$. The student should explain how to use the Distributive Property of Multiplication over

Addition to simplify this expression. Point out that they multiplied a monomial times a binomial when using this property.

- The third group's expression equals $x^2 + 9x + 18$. The student should explain how to use the FOIL Method to simplify the expression. Bring to their attention the intermediate step, $x^2 + 6x + 3x + 18$, and to review the meaning of FOIL (first, outer, inner, last).
- Tell students the methods used to simplify these expressions are similar to the methods they will learn in this lesson.

Section 1

Expand Their Horizons

In Section 1, students will apply the product properties of square roots and cube roots to multiply two monomials involving radical expressions. The Product Property of Square Roots states, for nonnegative numbers, the square root of the product is equal to the product of the square roots: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$. Similarly, the Product Property of Cube Roots states, for any real numbers, the cube root of the product is equal to the product of the cube roots: $\sqrt[3]{ab} = \sqrt[3]{a} \cdot \sqrt[3]{b}$.

Common Error Alert

Students may confuse the rules for adding radical terms with the rules for multiplying radicals. In order to simplify an expression when multiplying radicals, the radicals must have the same index but not necessarily the same radicand. Coefficients are multiplied to obtain a new coefficient, and radicands are multiplied to obtain a new radicand. Students may best understand "outside" numbers are multiplied together, and "inside" numbers are multiplied together. For example, $7\sqrt{3} \cdot 2\sqrt{3} = 7 \cdot 2\sqrt{3 \cdot 3} = 14 \cdot 3 = 42$. The expression $7\sqrt{3} + 2\sqrt{3}$, however, does not equal $(7 + 2)\sqrt{3 + 3}$. The expression $7\sqrt{3} + 2\sqrt{3}$ equals $9\sqrt{3}$.

Using these properties, students will learn, for a nonnegative number x, $\sqrt{x} \cdot \sqrt{x} = \sqrt{x \cdot x} = \sqrt{x^2} = x$. Initially, this concept may be confusing for students to understand. Illustrate this principle by giving an example such as $\sqrt{3} \cdot \sqrt{3} = \sqrt{3 \cdot 3} = \sqrt{3^2} = \sqrt{9} = 3$ or $\sqrt{5} \cdot \sqrt{5} = \sqrt{5 \cdot 5} = \sqrt{5^2} = \sqrt{25} = 5$.

When simplifying the product of radicals, each factor can be simplified before the multiplication operation. Consider the value $\sqrt{10} \cdot \sqrt{45}$. This expression can be written as $\sqrt{5} \cdot \sqrt{2} \cdot \sqrt{9} \cdot \sqrt{5}$. The value $\sqrt{5} \cdot \sqrt{5} = 5$ and $\sqrt{9} = 3$, so the expression $\sqrt{5} \cdot \sqrt{2} \cdot \sqrt{9} \cdot \sqrt{5}$ equals $15\sqrt{2}$.

Alternatively, use the Product Property of Square Roots to find the product of the two radicals and then, simplify. Multiply the radicands to form a new radicand. Then, if possible, factor the resulting radicand into a perfect square (or perfect cube) factor. The product of $\sqrt{10} \cdot \sqrt{45}$ is $\sqrt{450}$. This radical can be rewritten as $\sqrt{225} \cdot \sqrt{2}$, which equals $15\sqrt{2}$. This is the same as the result acquired by using the first method; either method will result in the correct answer.

1 Because both radicals are square roots, they can be multiplied using the Product Property or Square Roots. Multiply the coefficients seven and two to get 14. The radical $\sqrt{30}$ can be written as the product $\sqrt{6} \cdot \sqrt{5}$. The value $\sqrt{6} \cdot \sqrt{6} = 6$, so the expression becomes $14 \cdot 6\sqrt{5}$ or $84\sqrt{5}$. Using the alternative method for simplifying the product of radicals, $7\sqrt{30} \cdot 2\sqrt{6}$ becomes $14\sqrt{180}$. The radical term $\sqrt{180}$ can be written as $\sqrt{36} \cdot \sqrt{5}$, which simplifies to $6\sqrt{5}$. Again, the result is $14 \cdot 6\sqrt{5}$ or $84\sqrt{5}$.

2 Both factors are cube roots; they can be multiplied using the Product Property of Cube Roots. The value $\sqrt[3]{9k} \cdot \sqrt[3]{6k}$ equals $\sqrt[3]{54k^2}$. The only perfect cube factor other than one is 27, so this expression can be written as $\sqrt[3]{27} \cdot \sqrt[3]{2k^2}$, which is equivalent to $3\sqrt[3]{2k^2}$. This is in simplest form because two has no perfect cube factors other than one, and a radical with a variable cannot be reduced when the variable's exponent is smaller than the index of the radical. This problem could also be simplified by first factoring each radicand. The radical term $\sqrt[3]{9k}$ can be rewritten $\sqrt[3]{3} \cdot \sqrt[3]{3} \cdot \sqrt[3]{k}$, and $\sqrt[3]{6k}$ can be rewritten as $\sqrt[3]{3} \cdot \sqrt[3]{2k}$. The entire expression could be rewritten $\sqrt[3]{3} \cdot \sqrt[3]{3} \cdot \sqrt[3]{k} \cdot \sqrt[3]{3} \cdot \sqrt[3]{2k}$. This simplifies to $3 \cdot \sqrt[3]{k} \cdot \sqrt[3]{2k}$ or $3\sqrt[3]{2k^2}$.

Additional Examples

1. Simplify: $\sqrt[3]{-10x} \cdot \sqrt[3]{4x^2}$.

Multiply the radicands together, which gives $\sqrt[3]{-40x^3}$. The value –40 has the perfect cube factor –8; this expression can be written as $\sqrt[3]{-8}\sqrt[3]{5}\sqrt[3]{x^3}$ using the Product Property of Cube Roots. The cube root of –8 is –2, and $\sqrt[3]{x^3}$ equals x. This simplifies to $-2x\sqrt[3]{5}$.

2. Simplify: $\sqrt{x} \cdot \sqrt{x^3}$.

The radicand x^3 contains the perfect square factor x^2 because $x \cdot x^2 = x^3$. The radical expression $\sqrt{x} \cdot \sqrt{x^3}$ can be rewritten $\sqrt{x} \cdot \sqrt{x} \cdot \sqrt{x^2}$, using the Product Property of Square Roots. The expression $\sqrt{x} \cdot \sqrt{x}$ reduces to x, and $\sqrt{x^2}$ reduces to x. The simplified expression is $x \cdot x$ or x^2.

Section 2

Expand Their Horizons

In Section 2, the use of product properties to multiply monomials with radical expressions is extended to multiplication of a monomial times a binomial. The Distributive Property of Multiplication over Addition states that the product of a monomial times a binomial equals the sum of the products of the monomial and each of the terms of the binomial. Formally, for all real numbers a, b, and c, $a(b + c) = ab + ac$. After applying the Distributive Property, students will use what they have learned about the product of monomials to combine like terms and simplify.

3 Applying the Distributive Property yields $(\sqrt{6})(3) - (\sqrt{6})(\sqrt{3})$. The value $(\sqrt{6})(3)$ equals $3\sqrt{6}$. The product of $(\sqrt{6})(\sqrt{3})$ can be found by factoring six to obtain $(\sqrt{2})(\sqrt{3})(\sqrt{3})$. Since $\sqrt{3} \cdot \sqrt{3} = 3$, this becomes $3\sqrt{2}$. The value $(\sqrt{6})(\sqrt{3})$ also equals $\sqrt{18}$, which can be written as $\sqrt{9}\sqrt{2}$ using its perfect square factor. This simplifies to $3\sqrt{2}$. The value in simplest form is $3\sqrt{6} - 3\sqrt{2}$.

4 Using the Distributive Property, this can be written as $(\sqrt{2})(\sqrt{3}) - (\sqrt{2})(\sqrt{32})$. The first term, $(\sqrt{2})(\sqrt{3})$, equals $\sqrt{6}$ in simplest form. The second term, $(\sqrt{2})(\sqrt{32})$, can be written as $(\sqrt{2})(\sqrt{2})(\sqrt{16})$, where 16 is the largest perfect square factor of 32. The value $\sqrt{2} \cdot \sqrt{2}$ equals two and $\sqrt{16}$ equals four. So, $(\sqrt{2})(\sqrt{32})$ becomes eight. The value in simplest form is $\sqrt{6} - 8$.

Additional Examples

1. Simplify: $2\sqrt{3}(5 + \sqrt{3})$.

Using the Distributive Property, this expression becomes $2\sqrt{3} \cdot 5 + 2\sqrt{3} \cdot \sqrt{3}$. Since "outside" numbers are multiplied together, $2\sqrt{3} \cdot 5$ is $10\sqrt{3}$, and $\sqrt{3} \cdot \sqrt{3}$ is 3. The second term becomes $2 \cdot 3$ or six. The result is $6 + 10\sqrt{3}$.

2. Simplify: $\sqrt[3]{6}(\sqrt[3]{4} - \sqrt[3]{45})$.

Using the Distributive Property, this expression is $\sqrt[3]{6} \cdot \sqrt[3]{4} - \sqrt[3]{6} \cdot \sqrt[3]{45}$. Multiply the radicands together to produce $\sqrt[3]{24} - \sqrt[3]{270}$. A perfect cube factor of 24 is eight, and a perfect cube factor of 270 is 27. Using the Product Property of Cube Roots, the expression can be broken into factors as $\sqrt[3]{8} \cdot \sqrt[3]{3} - \sqrt[3]{27} \cdot \sqrt[3]{10}$. This yields $2\sqrt[3]{3} - 3\sqrt[3]{10}$ in simplest form.

Section 3

Expand Their Horizons

In Section 3, students will extend their knowledge of multiplication of a monomial with a binomial with radical expressions to multiplication of two binomials with radical expressions using the FOIL Method. The FOIL Method multiplies the first (F) term in each binomial, the outer (O) terms of the binomials, the inner (I) terms of the binomials, and then, the last (L) term in each binomial. All products are found by applying the Product Property of Square Roots or the Product Property of Cube Roots. Like terms should be combined, remembering like terms must have the same variable(s) with the same exponent(s) as well as the same radicand and index.

5 Rewrite the square as the product of two binomials: $(\sqrt{2} + \sqrt{6})(\sqrt{2} + \sqrt{6})$. Apply the FOIL Method. The product of the first terms $\sqrt{2} \cdot \sqrt{2}$ is two. The product of the outer terms $\sqrt{2} \cdot \sqrt{6}$ is $2\sqrt{3}$. The product of the inner terms $\sqrt{6} \cdot \sqrt{2}$ is $2\sqrt{3}$. In binomial squares the inner and outer products are always the same. Multiplication of the last terms $\sqrt{6} \cdot \sqrt{6}$ is six. After applying the FOIL Method, the square of the binomial expression is the sum of the products. This is $2 + 2\sqrt{3} + 2\sqrt{3} + 6$. Adding like terms yields $8 + 4\sqrt{3}$.

Remember when adding like radicals, only the coefficients are added, and the like index and radicand remain the same.

Common Error Alert

In simplifying a binomial square, students may mistakenly "distribute" the exponent two to each of the terms of the binomial. For instance, in Guided Notes Problem 5, students may incorrectly write $(\sqrt{2} + \sqrt{6})^2$ as $\sqrt{2}^2 + \sqrt{6}^2$. Encourage students who make this mistake to rewrite a square binomial as the product of the binomial times itself. Visually, this will remind students to apply the FOIL Method.

The square of a binomial expression equals the square of the first term, plus double the product of the first and second terms, plus the square the second term. This may be difficult for students to grasp. Using this method, in the previous problem, $(\sqrt{2} + \sqrt{6})^2$ equals $\sqrt{2}^2 + 2(\sqrt{2}\sqrt{6}) + \sqrt{6}^2$. After reducing radicals and combining like terms, the final result is the same, $8 + 4\sqrt{3}$.

6 Applying the FOIL Method, the product of the first terms is four. The product of the outer terms is $2(-\sqrt{x})$ or $-2\sqrt{x}$. The product of the inner terms is $2(\sqrt{x})$ or $2\sqrt{x}$. The product of the last terms is $\sqrt{x} \cdot (-\sqrt{x})$, which is $-x$. So, this yields $4 - 2\sqrt{x} + 2\sqrt{x} - x$. In combining like terms, the middle terms sum to zero. The final result is $4 - x$.

The previous problem is an example of the difference of squares. This expression is equivalent to the product of the first terms minus the product of the last terms. The middle terms cancel; it is not necessary that they be multiplied. In the previous example, $(2 + \sqrt{x})(2 - \sqrt{x})$ becomes $2^2 - \sqrt{x}^2$ or $4 - x$.

Look Beyond

Simplifying radicals means removing any perfect square (cube) factors from the radical and combining like terms. It also means removing radicals from the denominator of a fraction. One way in which radicals are removed from the denominator is by multiplying by its conjugate. Multiplying by the conjugate eliminates the radical in the denominator. This topic is covered in a future lesson.

Additional Examples

1. Simplify: $(\sqrt{3} + \sqrt{6})(2 + \sqrt{2})$.

Use the FOIL Method to multiply these binomials.

$\sqrt{3} \cdot 2 + \sqrt{3} \cdot \sqrt{2} + \sqrt{6} \cdot 2 + \sqrt{6} \cdot \sqrt{2}$

$2\sqrt{3} + \sqrt{6} + 2\sqrt{6} + 2\sqrt{3}$

$4\sqrt{3} + 3\sqrt{6}$

2. Simplify: $(3\sqrt{7} - 2\sqrt{5})(3\sqrt{7} + 2\sqrt{5})$.

This is a difference of squares.

$(3\sqrt{7})^2 - (2\sqrt{5})^2$

$3^2(\sqrt{7})^2 - 2^2(\sqrt{5})^2$

$9 \cdot 7 - 4 \cdot 5$

$63 - 20$

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