17.2 teacher notes

Objective

- Add and subtract radical expressions.

Vocabulary

Perfect square (Lesson 17-1)
Cube root (Lesson 17-1)
Index (Lesson 17-1)
Product Property of Cube Roots (Lesson 17-1)
Product Property of Square Roots (Lesson 17-1)

Prerequisites

Simplifying radicals

Adding and subtracting like terms

Get Started

- Lead a discussion of adding like terms. Ask, "What is 2 + 3?" Wait for students' reply of five.
- Now ask, "What is $2x + 3x$?" Wait for student reply of $5x$. Say, "What is $2x + 3$?" Wait for students to respond that these cannot be added because they are not like terms (one term contains a variable and the other a constant). Lead discussion, if necessary.
- Then ask, "What is $2y^2 + 3y^2$?" Wait for students' reply of $5y^2$. Follow up with the question, "What is $2y^2 + 3y$?" Wait for students' reply that these cannot be added because they are not like terms (having the same variable with the same power). Lead discussion, if necessary.
- Next ask, "What is 2♦ + 3♦?" (Use any symbol or figure to demonstrate the concept.) Wait for students' reply of 5♦. Follow up with the question, "What is 2♦ + 3♣?" Wait for students to reply that these are not like terms and cannot be added.
- Finally, pose the question, "What might $2\sqrt{7} + 3\sqrt{7}$ or $2\sqrt{7} + 3\sqrt{11}$ be?" Encourage students to write down what they think are the results and inform them they can check their answers at the end of the lesson. (The correct response is $2\sqrt{7} + 3\sqrt{7}$ is $5\sqrt{7}$, and $2\sqrt{7} + 3\sqrt{11}$ contains unlike terms that cannot be added.)

Section 1

Expand Their Horizons

In Section 1, students will combine radicals through addition and subtraction. Students will first have to determine if the radical terms are like terms. Adding radicals is the same as adding like terms. "Like" radical terms have the same radicand and the same index. Combine only like radical terms. Combining these like terms involves adding or subtracting their coefficients and keeping the radicals (i.e. the index and radicand) the same. Frequently, remind students expression with different radicands or different indices cannot be combined.

The usual meaning of coefficient is the numerical factor of a term as distinguished from a variable factor. In this lesson, coefficient means "any of the factors of a product considered in relation to a specific factor (the radical factor)." In this case, the coefficient is the nonradical factor.

1 The radicand is five and the index is two in each term. These are like radical terms and can be combined. Since $6 - 2 - 3$ is 1, then $6\sqrt{5} - 2\sqrt{5} - 3\sqrt{5}$ is $1\sqrt{5}$ or $\sqrt{5}$, where the coefficient is understood to be one.

2 In each term the radicand is four and the index is three. Since these are like terms, they can be combined. The coefficients are combined, but the radicals remain unchanged: $\sqrt[3]{4} - 2\sqrt[3]{4} + 5\sqrt[3]{4} = 1\sqrt[3]{4} - 2\sqrt[3]{4} + 5\sqrt[3]{4} = -1\sqrt[3]{4} + 5\sqrt[3]{4} = 4\sqrt[3]{4}$.

3 Rearrange the expression to group like radicals together. This yields $4\sqrt{5} - 3\sqrt{5} + 3\sqrt{6} + 2\sqrt{6}$. The first two terms are square roots with a radicand of five. Combine these terms to get $\sqrt{5}$. The last two terms are also square roots but with a radicand of six. Combine these terms to get $5\sqrt{6}$. The final result is $\sqrt{5} + 5\sqrt{6}$, which is in simplest form.

Common Error Alert

Stress to students that coefficients only are added or subtracted in expressions with like radical terms. Students may incorrectly try to add or subtract the radicals. For example, $5\sqrt{3} + 2\sqrt{3} \neq 7\sqrt{6}$; rather $5\sqrt{3} + 2\sqrt{3} = 7\sqrt{3}$. It may be helpful to compare this expression with $5x + 2x = 7x$.

Additional Examples

1. Simplify: $5\sqrt[3]{2} - 7\sqrt{2} - \sqrt[3]{2} + 4\sqrt{2}$.

The radicand of each term is two; however, the indices are different. Only those radical terms with the same radicand and the same index can be combined. The coefficients with the radicand remaining the same are added or subtracted in these like terms. Subtract $5\sqrt[3]{2} - \sqrt[3]{2}$ to get $4\sqrt[3]{2}$ and add $-7\sqrt{2} + 4\sqrt{2}$ to get $-3\sqrt{2}$. The result is $4\sqrt[3]{2} - 3\sqrt{2}$.

2. Simplify: $\frac{5}{6}\sqrt{7} - \frac{3}{4}\sqrt{7}$.

Since the radicands (7) and the indices (2) are the same, combine these radicals. Because the coefficients are fractions, the least common denominator (LCD) must be found in order to subtract. The LCD is 12. The coefficient $\frac{5}{6}$ can be written as $\frac{10}{12}$, and the coefficient $\frac{3}{4}$ can be written as $\frac{9}{12}$. The expression $\frac{5}{6}\sqrt{7} - \frac{3}{4}\sqrt{7}$ can be rewritten as $\frac{10}{12}\sqrt{7} - \frac{9}{12}\sqrt{7}$ which yields $\frac{1}{12}\sqrt{7}$.

Section 2

Expand Their Horizons

In Section 2, students will simplify radicals individually in order to obtain like radical terms, which are then combined. When simplifying, students will utilize the Product Property of Square Roots or the Product Property of Cube Roots. After simplifying, radicals must have the same radicand and the same index in order to be combined through addition or subtraction.

4 Each radical term must be simplified individually using the Product Property of Square Roots. This will help determine if the radical terms are like terms that can be added. The first term, $-\sqrt{20}$, can be written as $-\sqrt{4}\sqrt{5}$. Because $-\sqrt{4}$ can be simplified to –2, the radical term $-\sqrt{20}$ simplifies to $-2\sqrt{5}$. The second term is already simplified because five has no perfect square factor other than the number one. The last term, $\sqrt{80}$, can be written as $\sqrt{16}\sqrt{5}$. Because 16 is a perfect square factor of 80, $\sqrt{80}$ simplifies to $4\sqrt{5}$. The index and radicand are the same for each term in the expression, which means all terms are like terms and can be combined by adding and subtracting the coefficients. The expression $-\sqrt{20} - 2\sqrt{5} + \sqrt{80}$ simplifies to $-2\sqrt{5} - 2\sqrt{5} + 4\sqrt{5}$, which simplifies to $0\sqrt{5}$ or zero.

Common Error Alert

When the coefficient of a radical is zero, the entire term is zero. Students may mistakenly write the solution to a problem such as $-2\sqrt{5} - 2\sqrt{5} + 4\sqrt{5}$ above as $\sqrt{5}$ instead of $0\sqrt{5}$. When the coefficient of a radical is zero, the entire term is zero.

Additional Examples

1. Simplify: $\sqrt[3]{-16} + \sqrt[3]{250} - \sqrt[3]{-81}$.

All indices are three, indicating cube roots, but the radicands are not identical. Each radical can be simplified individually, and then, the ability to combine like terms should be re-evaluated. Using the Product Property of Cube Roots with a perfect cube factor of each, the expression can be rewritten as the following: $\sqrt[3]{-8}\sqrt[3]{2} + \sqrt[3]{125}\sqrt[3]{2} - \sqrt[3]{-27}\sqrt[3]{3}$. Simplifying the radicals of perfect cubes yields $-2\sqrt[3]{2} + 5\sqrt[3]{2} - (-3)\sqrt[3]{3}$. The radical terms $-2\sqrt[3]{2}$ and $5\sqrt[3]{2}$ are like terms. Adding and subtracting the coefficients of like terms and keeping the common radicand the same gives $3\sqrt[3]{2} + 3\sqrt[3]{3}$.

2. Simplify: $\frac{2}{3}\sqrt{45} + \frac{1}{2}\sqrt{80}$.

Both radical terms are square roots, but the radicands are different. Simplify each radical term. Using the Product Property of Square Roots with perfect square factors of each, this expression can be written as the following: $\frac{2}{3}\sqrt{9}\sqrt{5} + \frac{1}{2}\sqrt{16}\sqrt{5}$. Taking the square roots of the perfect squares yields $\frac{2}{3} \cdot 3\sqrt{5} + \frac{1}{2} \cdot 4\sqrt{5}$. Multiplication of the coefficients yields $2\sqrt{5} + 2\sqrt{5}$, which can be simplified to $4\sqrt{5}$.

Section 3

Expand Their Horizons

In Section 3, students will apply the concepts of adding and subtracting radicals to radicals with variables in the radicand. The rules for adding and subtracting radicals that contain variables are the same as for those that do not contain variables. Only like radical terms can be combined. If like radical terms are not immediately obvious, first simplify each term to see if they can be expressed as like terms.

Common Error Alert

Remind students $\sqrt{x^2} = |x|$, but $\sqrt[3]{x^3} = x$. The square root of a number is positive; this means the principle root must be used. The opposite of the principle root is $-\sqrt{x}$; in which case, $-\sqrt{x^2} = -|x|$.

5 Using the Product Property of Square Roots, $\sqrt{12x^2}$ can be written as $\sqrt{4}\sqrt{3}\sqrt{x^2}$. Because $\sqrt{4} = 2$ and $\sqrt{x^2} = |x|$, this gives $2\sqrt{3}|x|$. Simplifying the second radical in the same manner, $\sqrt{27x^2}$ may be rewritten as $\sqrt{9}\sqrt{3}\sqrt{x^2}$ because nine is a perfect square factor of 27. This simplifies to $3\sqrt{3}|x|$. Both terms have a radical of $\sqrt{3}$ and both have the variable factor of $|x|$. Subtracting these simplified terms gives $-1\sqrt{3}|x|$ or $-\sqrt{3}|x|$.

Since multiplication is commutative, the order of the factors in an answer may differ.

The term $5\sqrt{3}x$ may also be written as $5x\sqrt{3}$. While both are mathematically correct, because $5\sqrt{3}x$ and $5\sqrt{3x}$ look similar but their meaning is quite different, $5x\sqrt{3}$ is the preferred notation. The generally accepted convention is that constants are first, followed by variables in alphabetical order, and then followed by grouping and calculation notation such as radicals, parenthesis, and absolute values. Within those grouping notations, the constants are again first followed by the variables in alphabetical order. Following are some examples:

$2z(x + y)$ is preferred over $(x + y) \cdot 2z$;
$4x^2y^3$ is preferred over $4y^3x^2$;
$2x\sqrt{3}$ is preferred over $2\sqrt{3}x$;
$\sqrt{5xyz}$ is preferred over $\sqrt{zx5y}$;
$\sqrt{3}|x|$ is preferred over $|x|\sqrt{3}$;
$|-2|\sqrt{x}$ is preferred over $\sqrt{x}|-2|$.

Look Beyond

The benefit gained in learning the techniques of this lesson is more indirect than direct. In integral calculus, the integral of a sum is the sum of the integral of each term. If each term is a radical expression, it may be possible to simplify the sum. In so doing, the number of integral calculations is reduced. The ability to manipulate radical expressions is an important technique in mathematics, physics, and engineering.

Additional Examples

1. Simplify:

$$x^2\sqrt[3]{81x^3} + x\sqrt[3]{24x^3} - x\sqrt[3]{192x^3}.$$

While all indices indicate cube roots, each term must be individually simplified to determine if the radicands are the same, so they may be combined. The first term, $x^2\sqrt[3]{81x^3}$, can be written as $x^2 \cdot \sqrt[3]{27} \cdot \sqrt[3]{3} \cdot \sqrt[3]{x^3}$. This can be simplified to $x^2 \cdot 3 \cdot \sqrt[3]{3} \cdot x$ or $3x^3\sqrt[3]{3}$. The second term, $x\sqrt[3]{24x^3}$, can be written as $x \cdot \sqrt[3]{8} \cdot \sqrt[3]{3} \cdot \sqrt[3]{x^3}$. This can be simplified to $x \cdot 2 \cdot \sqrt[3]{3} \cdot x$ or $2x^2\sqrt[3]{3}$. The last term, $x\sqrt[3]{192x^3}$, can be written as $x \cdot \sqrt[3]{64} \cdot \sqrt[3]{3} \cdot \sqrt[3]{x^3}$. This can be simplified to $x \cdot 4 \cdot \sqrt[3]{3} \cdot x$ or $4x^2\sqrt[3]{3}$. The entire expression can be written as $3x^3\sqrt[3]{3} + 2x^2\sqrt[3]{3} - 4x^2\sqrt[3]{3}$. Now all terms have like indices and like radicands, but the first term contains the variable x^3 while the other terms contain the variable x^2. Only the last two terms are like terms and can be combined. Subtracting the coefficients, while the variables, powers, and radicals remain the same, yields $3x^3\sqrt[3]{3} + 2x^2\sqrt[3]{3}$.

2. Simplify: $\sqrt{18x^3} + \sqrt{50x^3}$.

If the power of a variable radicand is equal to or higher than the index of the radical, then that radicand can be reduced. Using the Product Property of Square Roots, this expression equals $\sqrt{9}\sqrt{2}\sqrt{x^2}\sqrt{x} + \sqrt{25}\sqrt{2}\sqrt{x^2}\sqrt{x}$. The factor $\sqrt{x^2}$ is equivalent to $|x|$, as explained in Lesson 17-1. Therefore, after taking the square root of the perfect squares, the expression becomes $3\sqrt{2} \cdot |x|\sqrt{x} + 5\sqrt{2} \cdot |x|\sqrt{x}$. Placing the coefficients of each term together and the radicands of each term together, we have $3\sqrt{2x}|x| + 5\sqrt{2x}|x|$. These are like terms because the radicand $2x$ and the index is the same in each and because the variable $|x|$ is the same in each. The result is $8\sqrt{2x}|x|$.