16.4 teacher notes

Objectives

- Solve work problems.
- Solve uniform motion problems.

Vocabulary

Rational equation (Lesson 16-1)
Work equation
Work rate
Least common denominator (Lesson 1-3)
Common factors (Lesson 12-1)
Uniform motion equation
Cross multiplication (Lesson 16-2)

Prerequisites

Solving rational equations

Using formulas

Get Started

- Tell the class, "In this lesson, we are going to study word problems in which a rational equation must be solved—they involve *work* and *time*."
- Write on the board or overhead projector: "Henry can build a shed in 20 hours."
- Ask, "What fraction of the shed can Henry complete in one hour?" The correct response is $\frac{1}{20}$.
- Introduce the equation *work rate* $\times$ *time worked* $=$ *work done*. Students should see they used this equation when determining the portion of the shed Henry can build in one hour.

$$\frac{1\text{ shed}}{20\text{ hr}} \cdot 1\text{ hr} = \frac{1\text{ shed}}{20\ \cancel{\text{hr}}} \cdot 1\ \cancel{\text{hr}} = \frac{1}{20}\text{ shed}$$

- Say, "The rate of work is one shed per twenty hours. The time worked is one hour. Therefore, the work done in one hour is one-twentieth of a shed."
- Say, "In this lesson, we will learn how to write and solve equations for determining the answers to questions like these."

Section 1

Expand Their Horizons

In Section 1, students will solve work problems. In a work problem, two workers are completing a task. The first worker works at one rate, and the second worker works at another rate. Together, the workers work at a third rate. An equation can be written and solved to find the amount of time needed to complete the job at this third rate.

Before beginning the lesson, it may be helpful to review how to solve rational equations. Let students solve the equation $\frac{x}{2} + \frac{x}{10} = 1$. To solve, multiply each side of the equation by the least common denominator (LCD) of 2 and 10; this is 10. Multiplying both sides of the equation by 10 gives the equation $5x + x = 10$, or $6x = 10$. The solution is $x = \frac{5}{3}$.

Students will use a chart to record information given in the problem. When filling in the chart for the first example of the lesson, be sure students understand the unit of measurement for each column.

A *rate* is a fraction comparing two measurements of different units. Students should be quite familiar with rates, but it may be helpful to use common rates like miles per hour. A speed of 25 miles per hour can be written as the ratio $\frac{25 \text{ miles}}{1 \text{ hour}}$.

In the painting example, Ron's work rate can be written as the ratio $\frac{1 \text{ room}}{6 \text{ hours}}$. The ratio shows the mathematical expression of the idea "Ron can paint the room in six hours." The number in the "time" column is measured in hours. This number represents the amount of time worked by each painter. The time it will take for Ron and Kia to paint together is unknown, so the entry in each cell is "x." To determine the entry in the "work done" column, multiply the work rate by the time.

After the chart is complete, the next step is to write an equation representing the information in the chart. Point out that the entries in the "work done" column show the *fraction* of the work done by each painter. The expression $\frac{x}{6}$ shows the fraction of the job done by Ron in x hours, and $\frac{x}{9}$ shows the fraction of the job done by Kia in x hours. Because the total work done is one room, the equation is $\frac{x}{6} + \frac{x}{9} = 1$.

1 This problem differs from the example. Here, the hot water rate and the "working together" rate are known; the cold water rate is unknown. Because the job is completed in six minutes, write 6 in each of the "time" cells. The hot water rate is expressed as $\frac{1}{14}$ tub/minute. The cold water rate is an unknown quantity. If it takes the cold water faucet x minutes to fill the tub, then the cold water faucet rate is $\frac{1}{x}$ tub/minute. The fraction of work done by the cold water faucet is $\frac{6}{x}$; the fraction of work done by the hot water faucet is $\frac{3}{7}$. The sum of the parts done by each faucet must equal one, because one tub is filled, so the equation is $\frac{6}{x} + \frac{3}{7} = 1$. To solve, multiply both sides of the equation by $7x$ to get $42 + 3x = 7x$. Solve to find the solution $10.5 = x$. The rate of work for the cold water faucet is $\frac{1 \text{ tub}}{10.5 \text{ minutes}}$, so it would take the cold water faucet 10.5 minutes to fill the tub.

Common Error Alert

Students may scan the problem for rates and incorrectly place information in the table. In Guided Notes Question 1, they may write $\frac{1}{14}$ for the cold water rate of work and $\frac{1}{6}$ for the hot water work rate. Warn students that work problems can take different forms. Remind students to read each problem carefully and determine the unknowns before writing information in the table.

Section 2

Expand Their Horizons

In Section 2, students use the distance formula to solve problems. Students should be familiar with the formula $d = rt$. Show them how to divide both sides of the formula by r to get $\frac{d}{r} = t$. This version of the formula is useful when the distance and rate are known, but time is unknown. Also, show them how to divide both sides of the formula $d = rt$ by t to get $\frac{d}{r} = t$. This version of the formula is useful when the distance and time are known, but rate is unknown.

In the train example, students should be able to determine the rates and distances for the two trains directly from information in the problem. The time traveled by each train is unknown.

Remind students when two of the three variables of the equation $d = rt$ are known, they can be used to find a value or expression for the third variable. So, for Train A, the rate is r, and the distance traveled is 200 km. Using the formula $\frac{d}{r} = t$, the expression for time traveled by Train A is $\frac{200}{r}$. Use the same technique to write the expression $\frac{300}{r + 40}$ for the time traveled by Train B.

The phrase "In the same time" indicates that the time traveled by each train is the same. Equate the expressions $\frac{200}{r}$ and $\frac{300}{r + 40}$ and solve. Because each side of the equation contains a single fraction, solve by cross multiplication.

Remind students to go back to the problem to determine what to do with the answer $r = 80$. The variable r represents the rate of the Train A. The problem asks for the rate of Train A; therefore, the answer is 80 km/h.

2 This problem is similar to the example. Plane 1's rate is r. Plane 2 travels at a rate 80 mph slower than Plane 1, this rate is $r - 80$. Subtract 80 because "slower" means "less than." Use the distance traveled by each plane, the rate of each plane, and the equation $t = \frac{d}{r}$ to write expressions for each plane's traveling time. Set the expressions equal to each other because both planes fly for the same amount of time. Solve the equation $\frac{1,280}{r} = \frac{1,120}{r - 80}$ by cross multiplication.

Some students may assign the variable r to represent the speed of Plane 2 and write the expression $r + 80$ to represent the speed of Plane 1. This strategy is equally valid; however, such students must remember that their value for r is the speed of the Plane 2, and in order to find the rate of Plane 1, they must use the expression $r + 80$ or $560 + 80 = 640$.

3 This problem describes the motion of one object, Vince, instead of comparing the motion of two different objects. Tell students they should think of the "going" and "returning" trips as separate events, each with its own distance, rate, and time. The phrase "return trip" indicates that the distance of the "going" and "returning" trips are equal. Each leg of the trip is 40 miles. The "returning" rate is twice the "going" rate, so represent the rates as $2r$ and r, respectively. The sentence "The entire trip took five hours" indicates that the sum of the "returning" and "going" times is five hours. Write and solve the equation $\frac{40}{r} + \frac{20}{r} = 5$, so $r = 12$. Since r represents Vince's "going" rate of speed, his rate on the way to the beach was 12 mph.

Common Error Alert

Students may use the expressions for time inappropriately. For example, in Guided Notes Question 3, they may write and solve the equation $\frac{40}{r} = \frac{20}{r}$. Remind students to read the problem carefully for key words and phrases that indicate whether the expressions for time are to be added or set equal to each other.

Look Beyond

Many types of problems other than work and uniform motion problems can be solved using rational equations. For example, in physics, if a circuit contains two resistances wired in parallel, the total resistance R (measure in Ohms) in the circuit can be found using the formula $\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$. So, if a circuit's total resistance is 4 ohms, and it contains a 10 ohm and an x ohm resistance wired in parallel, x can be found by solving the equation $\frac{1}{4} = \frac{1}{10} + \frac{1}{x}$.

Additional Examples

1. Larry can unload Tom's truck in 45 minutes. It takes Tom 60 minutes to unload his truck. How long would it take both Larry and Tom to unload the truck?

	rate of work	time worked	work done
Larry	$\frac{1}{45}$	x	$\frac{x}{45}$
Tom	$\frac{1}{60}$	x	$\frac{x}{60}$

$$\frac{x}{45} + \frac{x}{60} = 1$$
$$180 \cdot \frac{x}{45} + 180 \cdot \frac{x}{60} = 180 \cdot 1$$
$$4x + 3x = 180$$
$$7x = 180$$
$$x = \frac{180}{7}$$

The time worked x is approximately 25.7 minutes.

2. Linda walked for 2 miles, then walked another 2 miles at twice the original rate. The entire walk took $1\frac{1}{2}$ hours. What was Linda's rate for the second two miles?

	distance	rate	time
First 2 miles	2	r	$\frac{2}{r}$
Second 2 miles	2	$2r$	$\frac{1}{r}$

$$\frac{2}{r} + \frac{1}{r} = 1\frac{1}{2}$$
$$2r \cdot \frac{2}{r} + 2r \cdot \frac{1}{r} = 2r \cdot \frac{3}{2}$$
$$4 + 2 = 3r$$
$$6 = 3r$$
$$2 = r$$

The rate for the second two miles was $2r$ or 4 miles per hour.