UEM16L03TN_61519 6/18/03 5:14 PM Page 59A

16.3 teacher notes

Objectives

- Determine whether a function is an inverse variation, identify the constant of variation, and write the equation.
- Solve problems using inverse variation.

Vocabulary

Constant of variation (Lesson 16-2)

Inverse variation

Prerequisites

Solving rational equations

Writing equations from given data

Get Started

- Say, "For a given distance, like from San Francisco to Seattle, the faster the rate of travel the less time it will take to make the trip."
- Say, "The distance from San Francisco to Seattle is 800 miles. If I take a plane between these two cities, it will take less time then if I were to drive."
- Let students know that this is an example of an inverse variation. As one quantity (speed) increases, the other quantity (time) decreases.
- Invite discussion among students to see if they can think of any scenarios where as one quantity increases, the other quantity decreases. After the lesson, determine whether the students' scenarios were examples of inverse variation. If they are, the product of one quantity to the other is a constant.

Section 1

Expand Their Horizons

In Section 1, students will learn about inverse variation functions. This is a function of the form $xy = k$, where k is a nonzero constant. If $k = 18$, then $x = 1$ and $y = 18$ is a solution to the equation. As x increases, y decreases proportionally because the product of x and y is the positive constant 18. For any nonzero value x, a value for y is uniquely determined. For example, if $x = 3$, then the equation is $3y = 18$. When you solve for y, the result is $y = 6$. The following table shows how y varies as x varies. Inverse variation functions are usually defined for positive values of the variables.

x	y	xy
1	18	18
1.5	12	18
2	9	18
5	3.6	18
10	1.8	18

When given a table of values, the value of one variable decreases as the other value increases does not necessarily mean the equation is an inverse variation. The following table does not represent an inverse variation because the product of x and y is not a constant.

x	y	xy
1	1	1
2	$\frac{1}{4}$	$\frac{1}{2}$
3	$\frac{1}{9}$	$\frac{1}{3}$
4	$\frac{1}{16}$	$\frac{1}{4}$

It may be helpful for students to use common formulas in explaining the concept of inverse variation. The distance formula $d = rt$ and the area of a rectangle formula $A = lw$ are two such formulas.

In this lesson the equation $xy = k$ is given. By the Division Property of Equality, $y = \frac{k}{x}$ because x cannot equal zero. As an example, let the inverse variation function be the distance formula $d = rt$. If the distance d is constant, then the rate r varies inversely as time t. Consider a trip from Seattle, WA to Boulder, CO; this is a distance of 1,300 miles. The equation is $1{,}300 = rt$. For a rate of travel of 50 mi/h the trip would take $t = \frac{1{,}300}{50} = 36$ h to complete. For a plane traveling at 325 mi/h the trip would take $t = \frac{1{,}300}{325} = 4$ h to complete. Notice that as the rate of travel increases, the time decreases proportionally. The product of the two variables r and t equals the constant of variation d, 1,300.

Common Error Alert

Given a table of values containing positive and negative numbers, students may think this cannot represent an inverse variation. Remind students the table represents an inverse variation problem if the product of the two variables is a constant.

1 Here the table of values contains positive and negative integers and decimal numbers. In each case, however, the product xy is 36. Work on each row individually to determine the constant of variation. The variable y is said to vary inversely as x.

2 The constant of variation k is 36 and the inverse variation function is $xy = k$. By the Transitive Property of Equality, $xy = 36$. This is the inverse variation function. Students should check the validity of this equation by substituting x and y values from the table into the equation.

3 This table of values does not represent an inverse variation because as x increases y does not decrease. Every inverse variation function must satisfy this condition. Secondly, the product of each xy pair is zero. The definition of an inverse variation requires that product xy is nonzero.

Common Error Alert

Students may reason the previous problem is an inverse variation because the product of each xy pair is the constant zero. Remind students this constant must be nonzero.

Additional Examples

1. Does the following table represent an inverse variation? If so, what is the constant of variation?

x	y	xy
–5	–4	
4	5	
8	2.5	

Yes, this table represents an inverse variation. The product of each xy pair is 20. This is the constant of variation.

2. The variable y varies inversely as x: y is 4 when x is 2. What is the value of x when y is 5?

$$xy = k$$
$$2 \cdot 4 = 8$$
$$5x = 8$$
$$\frac{5x}{5} = \frac{8}{5}$$
$$x = \frac{8}{5} \text{ or } 1\frac{3}{5}$$

The variable x is $1\frac{3}{5}$ when y is 5.

Section 2

Expand Their Horizons

In Section 2, students will apply their knowledge of inverse variation. Typical equations used for application problems are $d = rt$, $A = lw$, and $f = ma$. In each case, the variable on the left hand side of the equation is a nonzero constant. Tell students to look for key phrases like "rate varies inversely as time" or "mass varies inversely as acceleration" to identify inverse variation problems.

In the first example, students will solve an equation to determine the speed they must travel, in order, to make the trip in a certain amount of time. The term speed is synonymous with rate. The ordered pair (time, speed) is given; from this ordered pair, the constant of variation may be determined. With this information students can solve for the speed s.

4 In this problem, the ordered pair is (frequency, length). The constant of variation is the product of these two variables, frequency times length equals k. Because k is known and a new length is given, an equation may be written to determine the new frequency. Remind students that the new length is greater than the old length; this means the new frequency will be less than the old frequency.

5 This problem uses the equation, price per book times the number of books equals the total price. It may help some students to better understand the problem by understanding the relationship between the variables' units.

$$\frac{\$}{\text{books}} \cdot \text{books} = \frac{\$}{\cancel{\text{books}}} \cdot \cancel{\text{books}} = \$$$

So, the "rate" dollars per book times the number of books has its units in dollars. The method for solving this problem is identical to the previous problem.

Connections

In physics, the work equation $w = fd$ is defined as work equals force times displacement. Here, if the amount of work done on an object is constant, force varies inversely as displacement. The force f equals mass times acceleration. In the work equation, if f is a nonzero constant, work is said to vary directly as displacement.

Look Beyond

In general, an inverse variation has the form $xy = k$, where k is a nonzero constant. If the unit of measurement of x is a "rate" like dollars per pound and y is in pounds, then, x varies inversely as y when their product is constant. Given the equation $d = rt$, differential calculus provides the means by which the rate of change may be determined from this equation. The rate of change of distance with respect to time is velocity, and the rate of change of r, which is velocity, is acceleration. So, if acceleration is in m/s^2, then acceleration times seconds equals velocity. Acceleration varies inversely as time when velocity is constant.

Additional Examples

1. Mass varies inversely as acceleration. If a 3 lb object is accelerated at 12 mi/s^2 under a constant force *f*, how heavy is an object accelerated at 24 mi/s^2?

$ma = f$

$3 \cdot 12 = 36$

$24m = 36$

$\frac{24m}{24} = \frac{36}{24} = \frac{3}{2}$

$m = 1.5$

The object is 1.5 lb.

2. The speed of a car varies inversely as time. If a car travels at 45 mi/h for 1.5 h, about how long will the car travel at 50 mi/h to travel the same distance *d*?

$rt = d$

$(45)(1.5) = d$

$67.5 = d$

$50t = 67.5$

$\frac{50t}{50} = \frac{67.5}{50}$

$t = 1.35$

The car will travel 1.35 hours or about 1 hour 20 minutes.