15.2 teacher notes

Objectives

- Simplify rational expressions to simplest form.
- Identify rational expressions that are already in simplest form.

Vocabulary

Rational expression (Lesson 15-1)
Simplest form (Lesson 12-1)
Common factors (Lesson 12-1)
Factor (Lesson 12-1)
Cancel
Restricted value (Lesson 15-1)

Prerequisites

Factoring of monomials

Factoring by removing common factors

Factoring trinomials of the form $ax^2 + bx + c$

Factoring the difference of squares

Get Started

- Write the fraction $\frac{1,890}{93,555}$ on the board and ask students to write the fraction in simplest form. Give them several minutes to complete the task, asking them not to reveal their answers.
- Say, "Suppose I told you that 1,890 is the product of 3, 5, 7, and 18, and 93,555 is the product of 3, 9, 11, 15 and 21."
- Write $\frac{1,890}{93,555} = \frac{3 \cdot 5 \cdot 7 \cdot 18}{3 \cdot 9 \cdot 11 \cdot 15 \cdot 21}$ on the board. Ask, "How can writing the numerator and denominator as products help you determine the simplest form of the fraction?"
- Ask the class to help reduce the fraction. By canceling common factors, the equation equals $\frac{1,890}{93,555} = \frac{3 \cdot 5 \cdot 7 \cdot 18}{3 \cdot 9 \cdot 11 \cdot 15 \cdot 21} = \frac{3 \cdot 5 \cdot 7 \cdot (9 \cdot 2)}{3 \cdot 9 \cdot 11 \cdot (5 \cdot 3) \cdot (7 \cdot 3)} = \frac{2}{11 \cdot 3 \cdot 3} = \frac{2}{99}$.
- Say, "Just as we can use factors to help write a real number fraction in simplest form, we can use factors to identify and divide out common factors in an algebraic rational expression. In this lesson, we will write rational expressions in simplest form."

Section 1

Expand Their Horizons

In this lesson, students learn how to write rational expressions in simplest form. To accomplish this, the students will factor the numerator and denominator and divide out (or "cancel") common factors. Express the fraction in simplest form by finding the products of the remaining factors in the numerator and denominator. Students are already familiar with how to reduce a fraction. A fraction is completely reduced, or written in simplest form, when its numerator and denominator have no common factors.

To simplify an algebraic expression like $\frac{15a}{25a^2}$, write the numerator ($15a$) as a product of factors and the denominator ($25a^2$) as a product of factors. The result is $\frac{15a}{25a^2} = \frac{3 \cdot 5 \cdot a}{5 \cdot 5 \cdot a \cdot a}$. Using the Commutative Property of Multiplication, the factors can be rearranged such that $\frac{15a}{25a^2} = \frac{3 \cdot 5 \cdot a}{5 \cdot 5 \cdot a \cdot a} = \frac{5 \cdot a \cdot 3}{5 \cdot a \cdot 5 \cdot a}$. This is equivalent to the product $\frac{5}{5} \cdot \frac{a}{a} \cdot \frac{3}{5 \cdot a}$ or $1 \cdot 1 \cdot \frac{3}{5 \cdot a}$. Because there are no common factors in the numerator and denominator of $\frac{3}{5a}$, the rational expression is in simplest form.

1 To express the rational expression in simplest form, one should factor $7f^2 - 21f$ and $f - 3$. To factor $7f^2 - 21f$, remove the common factor $7f$ to get $7f(f - 3)$. The denominator cannot be factored further. The numerator and denominator have a common factor, $f - 3$, which can be divided out. So, the fraction is equivalent to $\frac{7f}{1}$ or $7f$ in simplest form.

In this section, the "negative one technique" is reviewed. To simplify $\frac{s - 4}{4 - s}$, factor out negative one from the numerator to get $\frac{-1(-s + 4)}{4 - s} = \frac{-1(4 - s)}{4 - s} = -1$. Some students may recognize a fraction of the form $\frac{a - b}{b - a}$ is always equivalent to negative one. This is because the numerator and denominator are opposites. Demonstrate to students that the expressions $a - b$ and $b - a$ are opposites by showing a real number example, like $3 - 5$ and $5 - 3$. The quotient of opposites is negative one.

2 Students are likely to factor out a two in the numerator and get $\frac{2(3 - x)}{x - 3}$. Students should recognize the expressions $3 - x$ and $x - 3$ are opposites and have a quotient of negative one. The resulting expression is $\frac{2(x - 3)(-1)}{x - 3}$ or $\frac{-2(x - 3)}{x - 3}$. The expression $(x - 3)$ cancels out of the numerator and denominator.

Common Error Alert

When multiplying a multi-term expression such as $(3 - x)$ by negative one, students may forget to multiply each term by negative one. For example, some students may think that $-1(3 - x)$ is equal to $(3 + x)$ instead of $(-3 + x)$. Remind students by the Distributive Property of Multiplication over Addition multiply negative one to each term within the parenthesis.

Additional Examples

1. Simplify: $\frac{10m^2n}{15m^4}$.

Write the prime factorizations and then, divide out common factors.

$$\frac{10m^2n}{15m^4} = \frac{2 \cdot 5 \cdot m^2 \cdot n}{3 \cdot 5 \cdot m^4} = \frac{2 \cdot 1 \cdot 1 \cdot n}{3 \cdot 1 \cdot m^2} = \frac{2n}{3m^2}$$

2. Simplify: $\frac{3m - 9}{18 - 6m}$.

Factor the numerator first by removing the common factor three. Factor the denominator by removing the common factor six. The factors $(m - 3)$ and $(3 - m)$ are opposites and have a quotient of negative one.

$$\frac{3m - 9}{18 - 6m} = \frac{3(m - 3)}{6(3 - m)} = \frac{3(-1)(3 - m)}{6(3 - m)} = \frac{3(-1) \cdot 1}{6 \cdot 1} = \frac{-3}{6} = -\frac{1}{2}$$

Section 2

Expand Their Horizons

In Section 2, the expressions in the numerator and denominator of the rational expression to be simplified are more complex. In these problems, one or both of the expressions are trinomials, and more than one factoring method may be required.

Review how to factor a trinomial of the form $ax^2 + bx + c$ before beginning. Remind students of the trial-and-error nature of the task but make them aware they can use existing expressions as a "hint" when factoring an expression. For example, when simplifying the expression $\frac{p - 5}{p^2 - p - 20}$, note that the numerator is in simplest form. When factoring $p^2 - p - 20$, it is a reasonable guess that one of the factors will be $p - 5$ (so that it will cancel with the $p - 5$ in the numerator).

So, $(p - 5)(p + 4)$ is an efficient first guess at the factorization of the denominator.

3 Each of the expressions requires two methods to factor completely. First, remove common factors to get $\frac{5m(3m^2 - m - 2)}{5m(9m^2 - 4)}$. At this point, suggest that students continue factoring the denominator first, because it contains the difference of squares and will most likely be easier to factor than the trinomial in the numerator: $\frac{5m(3m^2 - m - 2)}{5m(9m^2 - 4)} = \frac{3m^2 - m - 2}{(3m + 2)(3m - 2)}$. Analyze the factors in the denominator to see if one of the binomial factors in the denominator can be used in factoring the numerator. In this case, the numerator can be factored using the binomial $(3m + 2)$. The expression becomes $\frac{(3m + 2)(m - 1)}{(3m + 2)(3m - 2)}$. This reduces to $\frac{m - 1}{3m - 2}$.

Additional Examples

1. Simplify: $\dfrac{5 + h}{h^2 + 7h + 10}$.

Rewrite the denominator as a product of binomials: $(h + 5)(h + 2)$. In the numerator, by the Commutative Property of Addition, $5 + h = h + 5$. The quotient of the expression $(h + 5)$ in the numerator and denominator is one.

$$\frac{5 + h}{h^2 + 7h + 10} = \frac{5 + h}{(h + 5)(h + 2)} = \frac{1}{1 \cdot (h + 2)} = \frac{1}{h + 2}$$

2. Simplify: $\dfrac{16 - b^2}{3b^2 - 48}$.

The expressions $16 - b^2$ and $b^2 - 16$ are opposites. Some students may recognize this and eliminate several steps. First, factor three out of the expression in the denominator. Then, apply the "negative one technique" in the numerator.

$$\frac{16 - b^2}{3b^2 - 48} = \frac{(-1)(b^2 - 16)}{3(b^2 - 16)} = \frac{(-1) \cdot 1}{3 \cdot 1} = -\frac{1}{3}$$

Look Beyond

In a future lesson in this course, students will see how the method of dividing out common factors can be used to divide polynomials. In that lesson, every factor of the divisor (denominator) will divide out, so the simplified expression is a polynomial. That polynomial is the quotient of the numerator and denominator.

For example, to find the quotient of $x^2 + 4x + 3$ and $x + 1$, write the problem as a fraction and then, factor and simplify.

$$\frac{x^2 + 4x + 3}{x + 1} = \frac{(x + 3)(x + 1)}{x + 1} = \frac{(x + 3) \cdot 1}{1} = x + 3.$$

So, $(x^2 + 4x + 3) \div (x + 1) = x + 3$.