14.2 teacher notes

Objective

- Graph equations in the form $y = a(x - h)^2 + k$.

Vocabulary

Relation (Lesson 9-1)
Quadratic equation (Lesson 13-1)
Reflection
Vertex (Lesson 14-1)
Axis of symmetry (Lesson 14-1)
Translation
Parabola (Lesson 14-1)

Prerequisites

Applying order of operations

Evaluating expressions

Plotting points

Reflecting a graph across an axis

Translating a graph vertically and horizontally

Completing the square

Get Started

- Draw any type of graph in the first quadrant (upper right) only.
- Say, "What would this graph look like reflected across the y-axis?" Invite student discussion and allow students to demonstrate their ideas.
- Illustrate the correct answer by using a piece of transparency paper. Place the edge of the paper along the y-axis. Trace the graph drawn in the first quadrant onto the paper. Leaving the edge of the paper along the y-axis lift, the right edge of paper and place it in the second quadrant. Tape the transparency paper in place. Tell students that this is called a reflection across the y-axis.
- Repeat this process for the x-axis with another piece of transparency paper. Remind students that this is called reflection across the x-axis.

Section 1

Expand Their Horizons

In Section 1, students will inspect equations in the form $y = ax^2$ to determine the direction and width of the opening of a parabola. Starting with the equation $y = x^2$, students will use a table of values to graph the points from the table to see the effects large and small values of a have on the graph. Then students will observe how the graph changes if a is positive or negative. Throughout the lesson, students will formulate generalizations about their observations.

A happy face is a simple mnemonic tool for remembering the opening direction of graphs of quadratic equations. Remind students that a positive value for a in the equation $y = ax^2$ indicates a curve which opens upward; draw a happy face with plus symbols as eyes and an upward opening parabola as the mouth. Similarly, remind the students that a negative value for a indicates a curve which opens downward; draw a sad face with minus symbols as eyes and a downward opening parabola as the mouth.

It may be helpful to show students the graph of $y = -ax^2$ is a reflection of the graph of $y = ax^2$ across the x-axis. Use two tables of values for the equations $y = \frac{1}{2}x^2$ and $y = -\frac{1}{2}x^2$ to illustrate this point. Demonstrating to students that the y values of corresponding x values for the two graphs are opposites will refresh and reinforce the concept of reflection across the x-axis.

Common Error Alert

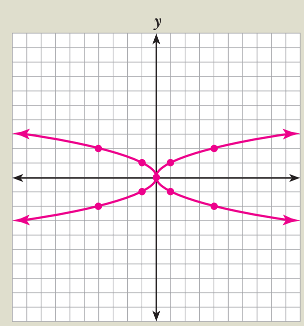
Students may confuse $-x^2$ with $(-x)^2$ when creating a table of values for the quadratic relation $y = -x^2$. Remind students $-x^2$ is equivalent to $-1x^2$, and the order of operations dictates the x-value be squared before it is multiplied by -1.

1 Graphs a and b open upward. Therefore, the coefficient a must be positive in the equations that correspond to both of these graphs. The equations $y = 1x^2$ and $y = 3x^2$ both have positive coefficients and must correspond to these graphs. As $|a|$ decreases in value, $y = ax^2$ gets wider. Since $|1|$ is less than $|3|$, the graph of $y = 1x^2$ is wider than the graph of $y = 3x^2$. The equation $y = 1x^2$ represents graph a, and the equation $y = 3x^2$ represents graph b. Graphs c and d open downward. Therefore, the coefficient a must be negative in the equations that correspond to both these graphs. The equations $y = -\frac{3}{4}x^2$ and $y = -5x^2$ both have negative coefficients and must correspond to these graphs. As $|a|$ decreases in value, $y = ax^2$ gets wider. Since $|-\frac{3}{4}|$ is less than $|-5|$, the graph of $y = -\frac{3}{4}x^2$ is wider than the graph of $y = -5x^2$. Thus, the equation $y = -\frac{3}{4}x^2$ represents graph c, and $y = -5x^2$ represents graph d.

Additional Examples

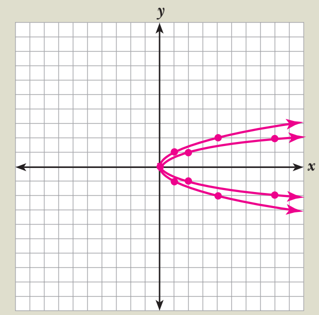
1. Compare the graphs of the quadratic equations $x = y^2$ and $x = -y^2$.

Create a table of values with –2, –1, 0, 1, and 2 as y values for $x = y^2$ and plot the points. Because there are two x values for one y value, the graph opens to the right but does not represent a function. Create a table of values for $x = -y^2$ using the same y values and draw the graph. The graph of $x = -y^2$ is the reflection of $x = y^2$ across the y-axis with $x = -y^2$ opening to the left. A positive a indicates the graph opens to the right, and a negative a indicates the graph opens to the left.



2. Compare the graphs of the quadratic relations $x = y^2$ and $x = 2y^2$.

Create a table of values and plot the points of each function. The graph of $x = y^2$ is a parabola opening to the right. The graph of $x = 2y^2$ also opens to the right, but it is narrower than $x = y^2$. Just as with parabolas of the form $y = ax^2$, the value $|a|$ indicates the width of the parabola. Since $|2|$ is greater than $|1|$, the parabola which represents $x = 2y^2$ is narrower.



Section 2

Expand Their Horizons

In Section 2, students will translate or shift parabolas horizontally, vertically, or both horizontally and vertically.

First, students examine equations in the form $y = ax^2 + k$. The value k represents vertical translation of the vertex. Positive values of k translate the vertex up, and negative values of k translate the vertex down. Since there is no horizontal translation in this equation, the vertex is $(0, k)$, and the axis of symmetry is $x = 0$.

2 The parabola $y = x^2$, which opens upward, is translated up $\frac{2}{3}$ unit. Therefore, the vertex is $\left(0, \frac{2}{3}\right)$, and the axis of symmetry is $x = 0$. In order to graph the equation, create a table of values with –2, –1, 0, 1, and 2 as the x values.

After putting quadratic equations in the form $y = a(x - h)^2 + k$, the students will learn the vertex of this equation is (h, k), with an axis of symmetry $x = h$.

Equations of the form $y = (x - h)^2$ will translate the parabola horizontally h units. The vertex of this equation is $(h, 0)$, and the axis of symmetry is $x = h$. Positive values of h translate the vertex to the right, and negative values of h translate the vertex to the left.

When creating a table of values, use h as the middle x value because it is the x-coordinate of the vertex. Choose a few consecutive integers greater than and less than h to complete the x values in the table. Using $y = (x - 5)^2$, $h = 5$. Possible x values in the table are 3, 4, 5, 6, and 7.

Common Error Alert

The equation $y = (x - h)^2$ may be confusing to students due to the subtraction symbol. Make sure the equation is written in this form and the value of h is subtracted from x. For example, $y = (x - 2)^2$ indicates $h = 2$. For $y = (x + 2)^2$, first write the equation as $y = (x - (-2))^2$. This indicates $h = -2$.

3 The vertex is $(h, 0)$ for equations of the form $y = (x - h)^2$. Since $y = (x + 3)^2$ can be written $y = (x - (-3))^2$, $h = -3$. The vertex is $(-3, 0)$, and the axis of symmetry is $x = -3$. Using h as the middle x value: –5, –4, –3, –2, –1, create a table of values for the graph. Use the order of operations in determining correct y values. Plot the points of the graph. Because all values are positive, the parabola opens up.

Vertical and horizontal translations are combined with opening width and direction in the vertex form of a quadratic equation,

$y = a(x - h)^2 + k$. Equations written in this form have the following properties:

- the vertex is (h, k).
- the axis of symmetry is $x = h$.
- the parabola opens up for positive a.
- the parabola opens down for negative a.
- the parabola gets narrower as $|a|$ increases.

An alternative form of a quadratic equation is $y - k = a(x - h)^2$. This form may be helpful for some students in determining the vertex of the equation. Set each side of the equation to zero. Solve for x and y using $y - k = 0$, which gives the y-coordinate of the vertex, and $a(x - h)^2 = 0$, which gives the x-coordinate of the vertex. For example, the equation $y - 3 = 2(x + 4)^2$ has a vertex of $(3, -4)$, which can be found by solving the equations $y - 3 = 0$ and $2(x + 4)^2 = 0$.

In order to put some quadratic equations in vertex form, it is necessary to complete the square. As an example, in the equation $y = x^2 - 10x + 24$, the student must complete the square to obtain the vertex (and equivalent) form $y = (x - 5)^2 - 1$. With the equation in this form, the student can identify the vertex and axis of symmetry. For quadratic equations with an x^2 coefficient other than positive one, remind students that the coefficient should be factored out when completing the square. (See Additional Examples.)

Common Error Alert

Once the form of the equation $y = (x - h)^2 + k$ is introduced, students trying to find the values of h and k may interpret an equation such as $y = x^2 + 3$ incorrectly. They may let $h = -3$. It may be helpful to remind students that 1 is the coefficient of x^2, but $h = 0$. The vertex form of the equation is $y = 1(x - 0)^2 + 3$.

4 This equation is in standard form. Complete the square to place the equation in vertex form. Move the constant term -3.2 to the left hand side of the equation. Take half of the coefficient of the x term ($-8x$) and square it. Half of -8 is -4 and $(-4)^2 = 16$. Add 16 to both sides and then, factor the right side of the equation into a perfect square $y + 19.2 = (x - 4)^2$. Isolate y to get $y = (x - 4)^2 - 19.2$. The vertex is $(4, -19.2)$.

Connections

The upper and lower supports of a bridge may be in the shape of a parabola. For instance, the Golden Gate Bridge in San Francisco, California, has parabolic supports that add not only to its aesthetic value but also to its stability. Spotlights and headlights are parabolic surfaces with a mirrored interior so that light not only shines outward but is also reflected outward parallel to the axis of symmetry. Buildings all over the world utilize parabolas as archways to provide support for their overall structure. Parabolic equations are used in aeronautics to describe the trajectory of objects and also used in business to predict the popularity and sales of a given product over time.

Look Beyond

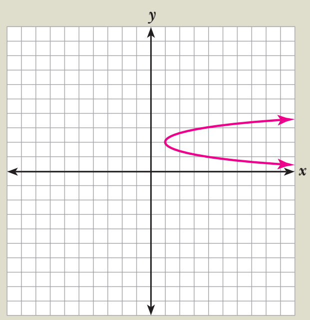
A parabola is an example of a conic section. In Algebra II and geometry, students will graph other conic sections with similar vertex forms: circles have the standard form $(x - h)^2 + (y - k)^2 = r^2$; ellipses have the standard form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$; and hyperbolas have the standard form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

In each case the point (h, k) represents a vital point of the graph such as the center of the circle. Also, the method of completing the square will be used to write standard form equations into **vertex** form for each of these conic sections.

Additional Examples

1. Find the vertex, axis of symmetry, and direction of opening for $x = 3(y - 2)^2 + 1$.

Since the y term is squared, this equation, which is not a function, opens sideways and has a horizontal line of symmetry. The value of a is 3, so it opens to the right. The vertex form of this equation is $x = (y - k)^2 + h$, where the vertex is (h, k) and the axis of symmetry is $y = k$. The vertex is (1, 2) because $h = 1$ and $k = 2$. The axis of symmetry is $y = 2$, which is the value of the y-coordinate of the vertex.



2. Find the vertex of $y = 2x^2 + 12x + 14$.

To find the vertex, write the equation in vertex form. Do this by completing the square. First, move the constant term to the left hand side of the equation. Then, factor out the coefficient of x^2.

$$y = 2x^2 + 12x + 14$$
$$y - 14 = 2x^2 + 12x$$
$$y - 14 = 2(x^2 + 6x)$$

To complete the square, take half of the coefficient of the linear term 6 and square it.

$$\frac{1}{2} \cdot 6 = 3$$
$$3^2 = 9$$
$$y - 14 + 18 = 2(x^2 + 6x + 9)$$
$$y + 4 = 2(x + 3)^2$$
$$y = 2(x + 3)^2 - 4$$

The vertex is (–3, –4) since $h = -3$ and $k = -4$.