14.1 teacher notes

Objectives

- Graph a quadratic relation by finding five to seven points on the graph.
- Graph a quadratic relation by finding the vertex and using symmetry.

Vocabulary

Relation (Lesson 9-1)
Function (Lesson 9-1)
Vertical line test (Lesson 9-1)
Quadratic relation
Standard form of a quadratic relation
Parabola
Vertex
Axis of symmetry
Symmetrical

Prerequisites

Applying the order of operations

Evaluating expressions

Plotting points

Applying the vertical line test to determine if a graph is a function

Get Started

- Separate the class into pairs.
- Instruct students to toss a beanbag or other soft object back and forth, making the trajectory an arc. Say, “The path of the beanbag is called a parabola.”
- Instruct students to throw the object so that the trajectory is higher or lower. Say, “The highest point reached by the beanbag is called the vertex.”
- Move the students closer together and continue to toss the object. Point out that the parabola is narrower. Move the students farther apart and point out that the parabola is wider.
- Tell the students that a quadratic relation contains two variables, one variable to the first power and the other variable to the second power. The graph of a quadratic relation is a U-shape called a parabola. Sketch four parabolas on the board to show the four directions a parabola can open up, down, right, or left. Teachers may want to demonstrate the four possibilities with their arms.

Section 1

Expand Their Horizons

In Section 1, students will represent quadratic relations graphically as parabolas. They will graph parabolas by plotting points. After graphing $y = x^2$ and $y = -x^2$, students will use the concept of symmetry to graph parabolas by plotting fewer points. Students will see that if a quadratic relation is a function, the vertex represents either the minimum or the maximum. Tossing a ball or an eraser to demonstrate a parabola is often helpful. Tell students that the trajectory (path) of an object can vary higher, lower, wider, or narrower, but every trajectory is a parabola.

The symmetrical property of parabolic graphs can be related to the Symmetric Property of Equality, which states that for any real numbers a and b, if $a = b$ then $b = a$. The axis of symmetry can be related to the equal symbol.

A table of values might be used to show the symmetry of points in a parabola. For an equation of the form $y = x^2 - 4x + 3$, the x-coordinate of the vertex is 2. If finding five points for the graph, this value will be placed as the middle (third) of five values in the x-column, so that the x values are 0, 1, 2, 3, and 4. The corresponding y values should be calculated for these values. Point out to the students that the first and fifth y values are the same, as are the second and fourth y values. After a few examples, a student will surely note that two of these calculations need not be made.

Common Error Alert

Students should understand that $-b$ in the axis of symmetry formula indicates the opposite of the coefficient b, which may be either positive or negative.

A memory aid for the opening direction of quadratic function graphs would be to remind the students that a positive a indicates a curve that opens up; draw a happy face with plus symbols as eyes and an upward opening parabola as the mouth. Similarly, remind the students that a negative a indicates a curve that opens down; draw a sad face with minus symbols as eyes and a downward opening parabola as the mouth. If students know the direction of the parabola opening before they begin their calculations, they will be more likely to find their own mistakes. This topic is studied in detail in Lesson 14-2.

1 The first task is to isolate the y to get the equation in standard form. Note that this equation has no x term, so standard form for this equation is $y = 1x^2 + 0x - 8$. So, $a = 1$ and $b = 0$, and $\frac{-b}{2a} = \frac{-0}{2(1)} = 0$. Therefore, the equation of the axis of symmetry is $x = 0$.

Common Error Alert

Students may assume that b in the axis of symmetry formula is the second term or the term following the x^2 term. This is the case only for a quadratic relation that is in standard form with three terms. Students should understand that b is the coefficient of the x term and that a missing linear term indicates that b is zero.

The standard form for a quadratic relation that is not a function is $x = ay^2 + by + c$, and the equation of the axis of symmetry is $y = -\frac{b}{2a}$. The graph of a quadratic relation of this type is a parabola that opens left or right.

Common Error Alert

Students should place the x value as the first coordinate in an ordered pair and the y value as the second coordinate in the ordered pair. It is easy to make the mistake of placing the first value found in the first location.

Connections

Some business applications involve quadratic functions to represent cost, revenue, or profit. It is useful to find minimum cost, maximum revenue, and maximum profit. In physics, the height of a freely falling object or projectile is modeled by a quadratic function.

Look Beyond

In more advanced algebra courses, students will graph relations in which both variables have a power of two. These graphs will form various curves called circles, ellipses, and hyperbolas. Parabolas, circles, ellipses, and hyperbolas are the various types of conic sections. The concept of symmetry will be helpful for all these curves.

In Lesson 14-2, students will learn to identify the shape (narrow or wide), orientation (up, down, left, or right), and location in the coordinate plane of a parabola by analyzing its equation.

In Lesson 14-3, students will solve application problems using the graphs of quadratic relations to find maximum and minimum values. In calculus, students will find maximum and minimum values in equations of third, fourth, and higher degrees.

Additional Examples

1. Find the axis of symmetry for the quadratic relation $y = \frac{1}{2}x^2 - 3x$.

Ask students to identify a (the coefficient of the quadratic term) and b (the coefficient of the linear term). Use the equation for the axis of symmetry, $x = -\frac{b}{2a}$.

$x = \frac{-(-3)}{2\left(\frac{1}{2}\right)} = \frac{3}{1} = 3$

Thus, $x = 3$ is the axis of symmetry for this quadratic relation.

2. Find the vertex for the quadratic relation $x = -2y^2 + 6y - 4$.

This equation is in the form $x = ay^2 + by + c$. The axis of symmetry has equation $y = -\frac{b}{2a}$.

$y = \frac{-6}{2(-2)} = \frac{-6}{-4} = \frac{6}{4} = \frac{3}{2}$. Substitute $\frac{3}{2}$ for y in the original equation.

$$\begin{aligned} x &= -2\left(\tfrac{3}{2}\right)^2 + 6\left(\tfrac{3}{2}\right) - 4 \\ &= -2\left(\tfrac{9}{4}\right) + 6\left(\tfrac{3}{2}\right) - 4 \\ &= -\tfrac{9}{2} + 9 - 4 \\ &= \tfrac{1}{2} \end{aligned}$$

The vertex is $\left(\frac{1}{2}, \frac{3}{2}\right)$.