13.5 teacher notes

Objectives

- Solve quadratic equations in one variable using the quadratic formula.
- Use the discriminant to determine the number of solutions of a quadratic equation in one variable.

Prerequisites

Solving quadratic equations by evaluating square roots

Solving quadratic equations by completing the square

Vocabulary

Standard form of a quadratic equation (Lesson 13-1)
Quadratic equation (Lesson 13-1)
Quadratic formula
Discriminant

Get Started

- Instruct students to solve $x^2 - 64 = 0$. Ask students what method they used to solve the equation: evaluating square roots, factoring, or completing the square. Write on the board the total number of students using each method. The solutions are $x = 8$ and $x = -8$.
- Repeat the process with $x^2 + 6x + 9 = 0$, $x^2 - 7x - 8 = 0$, and $x^2 - 2x + 2 = 0$. The solution of $x^2 + 6x + 9 = 0$ is $x = -3$. The solutions of $x^2 - 7x - 8 = 0$ are $x = 8$ or $x = -1$. The equation $x^2 - 2x + 2 = 0$ has no solution.
- All of the students should have used the completing the square method to solve the equation $x^2 - 2x + 2 = 0$. Point out to students that any quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, can be solved by completing the square. Likewise, any quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, can be solved by using the quadratic formula, which is the focus of today's lesson.

Section 1

Expand Their Horizons

In Section 1, students will be given the quadratic formula. The quadratic formula is derived by completing the square for the equation $ax^2 + bx + c = 0$, $(a \neq 0)$ as follows:

$ax^2 + bx + c = 0, (a \neq 0)$	Given.
$ax^2 + bx = -c$	Subtract c from both sides.
$x^2 + \frac{bx}{a} = \frac{-c}{a}$	Divide both sides by a.
$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$	Complete the square.
$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$	Factor the left side. Simplify the last term on the right.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Rewrite with a common denominator.
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	Evaluate square roots.
$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$	Subtract $\frac{b}{2a}$ from both sides.
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$	Rewrite the square root of the quotient as the quotient of square roots.
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	Simplify the second denominator.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Add fractions.

Common Error Alert

Make sure that the students realize that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is a shorthand way of writing the two solutions, $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

1 **Solve by using the quadratic formula. $3x^2 + 6x + 3 = 0$** In the lesson, the equation is solved using the quadratic formula with $a = 3$, $b = 6$, and $c = 3$. However, $3x^2 + 6x + 3 = 0$ may be solved by first dividing every term in the equation by three to get $x^2 + 2x + 1 = 0$. For the equation $x^2 + 2x + 1 = 0$, $a = 1$; $b = 2$; and $c = 1$. So, $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(1)}}{2(1)} = \frac{-2 \pm \sqrt{4 - 4}}{2} = \frac{-2 \pm \sqrt{0}}{2} = \frac{-2}{2} = -1$. Because both adding and subtracting zero to negative two is negative two, there is only one solution. This will happen any time the radicand is zero.

In the lesson, the solution to $3x^2 + 6x + 3 = 0$ is shown as $x = -1$ or $x = -1$. In cases such as this, $x = -1$ is called a multiple root because $3x^2 + 6x + 3 = 0$ is equivalent to $3(x - 1)(x - 1) = 0$. Moreover, it can be said that $x = -1$ is a root of multiplicity two or that $x = -1$ is a double root.

Common Error Alert

Students may incorrectly identity a, b, and c, when a quadratic equation is not in standard form. Make sure students rewrite the equation in standard form and write the values of a, b, and c before they use the formula.

Additional Examples

1. Solve: $x^2 - 17 = 0$.

$a = 1, b = 0, c = -17$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-0 \pm \sqrt{0 - 4(1)(-17)}}{2(1)}$$

$$x = \frac{\pm\sqrt{(4)(17)}}{2} = \frac{\pm 2\sqrt{17}}{2} = \pm\sqrt{17}$$

Solution set: $\{\sqrt{17}, -\sqrt{17}\}$

2. Solve: $x^2 + 5x = -8$.

Rewrite the equation in standard form.

$x^2 + 5x + 8 = 0.$

$a = 1, b = 5, c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{25 - 4(1)(8)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 32}}{2} = \frac{-5 \pm \sqrt{-7}}{2}$$

There is no real solution because $\sqrt{-7}$ is not a real number.

Section 2

Expand Their Horizons

In Section 2, the discriminant of a quadratic equation is defined as the expression $b^2 - 4ac$, the radicand in the quadratic formula. The discriminant of a quadratic equation will allow students to determine how many real number solutions there are to the equation.

If the discriminant is positive, there are two real solutions, $x = \frac{-b+\sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b-\sqrt{b^2 - 4ac}}{2a}$. If the discriminant is positive and a perfect square, the two solutions are rational, and the quadratic equation could be solved by factoring. If the discriminant is zero, the quadratic equation has one real solution, $x = \frac{-b}{2a}$. If the discriminant is negative, the quadratic equation has no real solutions because the square root of a negative number is not a real number. It will have two complex solutions.

2 **Use the discriminant to determine the number of solutions to $x^2 - 7x - 10$** The discriminant $b^2 - 4ac = 49 - 4(1)(-10)$ or 89. Because the discriminant is positive, the equation will have two real solutions. The two solutions may be considered the zeros of the quadratic function $y = x^2 - 7x - 10$. A zero of a function is the x-value of the x-intercept of the graph of the function. The graph of $y = x^2 - 7x - 10$ is a parabola that intercepts the x-axis in two points.

3 **Solve by using the quadratic formula. $x^2 - 7x - 10$** Substitute the values of 1, –7, and –10 for a, b, and c in the quadratic formula. The solutions are $x = \frac{7 + \sqrt{89}}{2}$ and $x = \frac{7 - \sqrt{89}}{2}$. Note that approximate values of these solutions can be found by taking the square root of 89. The solutions, to the nearest hundredth, are 8.22 and –1.22.

To solve the equation $x^2 - 7x - 10 = 0$ by using a graphing calculator, do the following:

1. Enter the quadratic expression $x^2 - 7x - 10$ into the calculator using the Y= button. **(Figure 1)**
2. Graph the function: press ZOOM 6 to use the standard window. This window does not show enough of the graph. Press WINDOW and use ▼ to set Ymin = –40 and Ymax = 40.
3. Press GRAPH.
4. Select CALC by pressing 2nd TRACE.
5. Select **2:zero** by pressing 2. The current graph is displayed with **LeftBound?** in the bottom-left corner.
6. Press ◄ or ► to select the x-value for the left bound of an interval in which the negative solution lies; then, press ENTER. A ► indicator on the graph screen shows the left bound. In the bottom left corner of the screen, **RightBound?** is displayed. Press ◄ or ► to select the x-value for the right bound of an interval in which the negative solution lies; then, press ENTER. A ◄ indicator on the graph screen shows the right bound. **Guess?** is displayed at the bottom-left corner. **(Figure 2)**
7. Press ◄ or ► to select a point between the bounds, near the zero of the function and then, press ENTER.
8. **Zero**
   X = –1.216991 Y = 0
   is displayed in the bottom-left corner. **(Figure 3)**
9. Repeat Steps 4–7 to find the other solution.
   Zero
   X = 8.2169906 Y = 0
   is displayed in the bottom-left corner.

Figure 1

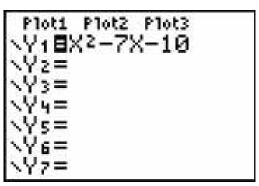


Figure 2

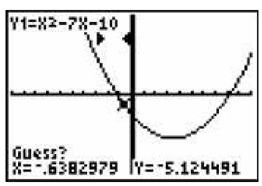
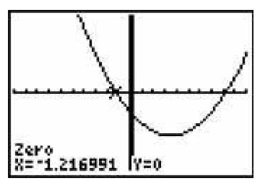


Figure 3



Any polynomial equation can be solved using the **zero** function of the calculator. If a polynomial expression is equal to zero, students can replace zero with y to create a corresponding polynomial function.

For example, the solutions of the polynomial equation, $x^3 + x^2 - 8x - 3 = 0$, are the zeros, or x-intercepts, of the function $y = x^3 + x^2 - 8x - 3$. The same method used for finding the zeros of the quadratic equation may be applied to any polynomial equation. **(Figure 4)**

Notice this third degree polynomial function has three zeros. **(Figure 4)** In general, an nth degree polynomial function can have at most n zeros.

Figure 5

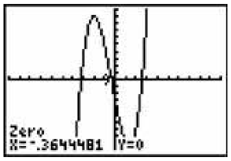
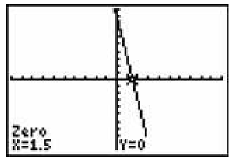


Figure 6



The equation $-6x + 9 = 0$ has the corresponding function $y = -6x + 9$. The graph of this function is a line; therefore, the function is a linear function. Be sure to contrast the graph of a quadratic function, which is a parabola, with the graph of a linear function, which is a line. The zero, or x-intercept, of this line is 1.5. **(Figure 5)**