13.1 teacher notes

Objectives

- Review standard form of a linear equation.
- Identify equations as quadratic, linear, or neither.
- Write quadratic equations of one variable in standard form.
- Identify the values of *a*, *b*, and *c*.

Vocabulary

Linear equation
Polynomial equation
Degree of a polynomial (Lesson 2-1)
Quadratic equation
Standard form of a linear equation
Standard form of a quadratic equation

Prerequisites

Applying laws of exponents

Combining like terms

Applying the Distributive Property

Rewriting equations using properties of equality

Recognizing linear equations of one variable

Get Started

- Divide the class into groups of three or four students. Provide each group with a set of cards having symbols $+$, $-$, $\cdot$, $=$, x, x^2, and single digit numerals. 3×5 cards cut in half work well. There should be several cards of each symbol and each numeral but only one equal sign card.
- The groups should use the cards to create linear equations. For example, $3x + 4 = 0$ would look like

  $\boxed{3}\ \boxed{x}\ \boxed{+}\ \boxed{4}\ \boxed{=}\ \boxed{0}$

- Next, the groups should use the cards to create quadratic equations. For example, $x^2 + 4x - 5 = 0$.

- Encourage students to create equations with several terms on each side; for example, $2x^2 + 5 = 5x + 8$. The equations need not be simplified or put into standard form during this activity.
- The students should then write equations for other groups to show using the cards.
- Tell the class the new lesson concerns how to determine whether an equation is linear or quadratic.

Section 1

Expand Their Horizons

In this lesson, students will learn to recognize quadratic equations. A quadratic equation in one variable is an equation which can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$. If an equation cannot be written in this form, it is not a quadratic equation.

A linear equation in one variable is an equation in which the greatest power of the variable is one. Every linear equation in one variable can be written in the form $ax + b = 0$, where $a \neq 0$. If an equation cannot be written in this form, it is not a linear equation.

It will be helpful to review the definitions of monomial and polynomial before beginning the lesson. A monomial is a constant, a variable, or the product of constants and variables; a polynomial is a monomial or the sum of monomials. A polynomial equation is an equation in which the expressions on both sides of the equation are polynomials. The equation $4x + 3 = -x^4 - 3x + 1$ is a polynomial equation since its left side is the binomial $4x + 3$, and its right side is the trinomial $-x^4 - 3x + 1$.

Any polynomial equation can be written in standard form. A polynomial equation in standard form has zero on one side of the equation and a polynomial in decreasing degree of the variable on the other side. That is, the exponents in the polynomial decrease from left to right. An example of a polynomial equation in standard form is $4x^5 + 3x^4 + x^2 - 1 = 0$. It does not matter which side of the equation the zero appears, although it may be easier for students to solve quadratic equations (in later lessons) if they put the zero on whichever side will make the squared term positive.

Writing polynomial equations in standard form is useful for several reasons. One reason is it makes it easier to identify whether the equation is linear (of degree 1), quadratic (of degree 2), or of higher degree.

A quadratic equation is in standard form when it is written in the form $ax^2 + bx + c = 0$. Discuss the restriction $a \neq 0$. If $a = 0$, the equation is in the form $0x^2 + bx + c = 0$ or $bx + c = 0$, which is a linear equation. In a quadratic equation in standard form, b and/or c can be zero. The equation $x^2 = 5$ can be written as a quadratic equation in standard form by subtracting five from both sides to get $x^2 - 5 = 0$. If the equation is written $1x^2 - 0x - 5 = 0$, it is easy to identify the values of a, b, and c; $a = 1$, $b = 0$, and $c = -5$.

Sometimes, linear and quadratic equations are in forms other than their standard forms. To determine whether an equation is linear, quadratic, or neither, rewrite it in standard form. Students may be surprised that the word *quadratic* indicates a power of two rather than four. They have seen many words based on the root *quad-*. Everyday words such as quadruplet (one of **four** children from a single birth) and quadruped (a **four**-footed animal), as well as mathematical terms like quadrilateral (a **four**-sided figure) and quadrant (one of **four** regions of the coordinate plane) all indicate the number four. Point out the Latin word for *square* is *quadratus* and this term leads both to the word quadratic, and to the words in which the prefix quad- indicates four since a square has four sides.

1 **Is the equation $8^2f + 2f = -9$ a quadratic equation, a linear equation, or neither?** In any equation written in standard form, one side of the equation is equal to zero. To write this equation in standard form, add nine to both sides to get $8^2f + 2f + 9 = 0$. Next, simplify the expression on the left side of the equation to get $64f + 2f + 9 = 0$; then $66f + 9 = 0$. Since the highest power of the variable f is one, this is a linear equation. The coefficient of the linear term, a, is 66; the constant term, b, is nine.

Common Error Alert

Students may see the power of two in the original equation and decide that the equation is quadratic. Remind them that the degree of the equation depends on the degree of the *variable*.

2 **Determine whether the equation $z^2 = 6$ is quadratic, linear, or neither.** Subtract six from both sides of the equation to get $z^2 - 6 = 0$. Inserting the term $0z$ does not change the value of the expression on the left side of the equation but does help the student identify the values of a, b, and c.

3 **Determine whether the equation $x^2 - 4x = x^2 - 2x + 1$ is a quadratic equation, a linear equation, or neither.** Since the term x^2 appears on each side of the equation, it can be subtracted from each side to get the equivalent equation $-4x = -2x + 1$. In this case, since there is only one term on the left side of the equation and there are two terms on the right side, some students may find it more efficient to add $4x$ to both sides of the equation to get $0 = 2x + 1$. This equation is equivalent to the equation in the DVD, $-2x - 1 = 0$.

It can be verified the equations are equivalent by showing that they have the same solution, $-\frac{1}{2}$.

As illustrated in the discussion above, if the sign of every term in an equation is changed, the resulting equation is equivalent. For example, $2x + 1 = 0$ is equivalent to $-2x - 1 = 0$ because they have the same solution, $-\frac{1}{2}$. $x^2 - 8 = 1$ is equivalent to $-x^2 + 8 = -1$ because they have the same solutions, 3 and -3.

4 **Is the equation $d^2(d + 4) = 0$ quadratic, linear, or neither?** The right side of the equation is already zero; the left side of the equation needs to be simplified before its degree can be determined. Distribute d^2 to the binomial $d + 4$ to get $d^3 + 4d^2 = 0$. Since the highest power of the variable d is three, this equation is neither linear nor quadratic. If students are curious, let them know that this is a *cubic* equation.

Look Beyond

In future lessons in this module, students will learn several methods for solving quadratic equations. When solving quadratic equations by factoring, it is necessary to have all nonzero terms on one side of the equation. When solving quadratic equations using the *quadratic formula*, the values of *a*, *b*, and *c* are substituted into a formula to get the solutions. It is essential to properly identify the values of *a*, *b*, and *c*, a task which requires putting the equation into standard form. Note that it may be easier for students to solve quadratic equations if they put the zero on whichever side will make the squared term positive.

Additional Examples

1. Determine whether the equation is quadratic, linear, or neither.

$$4b - 2^3 + (2b)^2 = 5$$

Simplify the left side of the equation and then, subtract five from both sides.

$$4b - 2^3 + (2b)^2 = 5$$
$$4b - 8 + 4b^2 = 5$$
$$4b - 8 \mathbf{- 5} + 4b^2 = 5 \mathbf{- 5}$$
$$4b - 13 + 4b^2 = 0$$
$$4b^2 + 4b - 13 = 0$$

Quadratic

2. Determine whether the equation is quadratic, linear, or neither.

$$2 = 4 - r(3 - r)$$

Simplify the right side of the equation and then, subtract two from both sides.

$$2 = 4 - r(3 - r)$$
$$2 = 4 - 3r + r^2$$
$$2 \mathbf{- 2} = 4 \mathbf{- 2} - 3r - r^2$$
$$0 = 2 - 3r + r^2$$
$$0 = r^2 - 3r + 2$$

Quadratic

Manipulatives

Algebra tiles can be used to model a linear or quadratic equation. Then, by adding tiles appropriately, the standard form of that equation can be modeled. To model an equation, use the tiles to model the expressions on each side of the equation, placing each expression on a different side of the equation mat. To model the standard form of the equation, add tiles to one side of the mat so that every original tile on that side is paired with its opposite tile. (In this discussion, we will add to the right side of the mat; the zero pairs that result on the right side will represent zero). For example, if there is a blue x^2 tile on the right side, add a red x^2 tile; this pair equals zero and can be removed. For every tile added to the right side, add an identical tile to the left side. Remove all zero pairs from the right side. This will result in the right side of the mat being empty, representing zero. It may be advantageous to work with just on type of tile at a time; that is, 1's tiles first, then x tiles, then x^2 tiles.

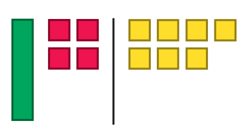
When one side of the equation mat is empty, the polynomial equation is in standard form. A polynomial equation is linear if at least one x tile remains and no x^2 tile remains. A polynomial equation is quadratic if at least one x^2 tile remains.

Next, students should model the equations below and put them in standard form to determine whether each is a linear equation, a quadratic equation, or neither.

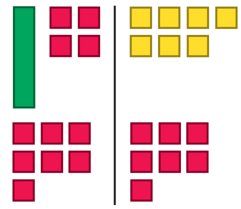
Example 1: $x - 4 = 7$

Answer: $x - 11 = 0$; linear

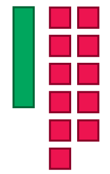
Model the equation by putting a green (positive) x tile and four red (negative) 1's tiles on the left side of the mat. Put seven yellow (positive) 1's tiles on the right side.



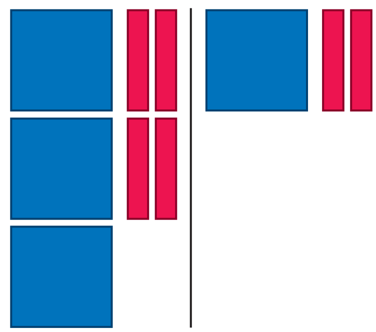
Add seven red (negative) 1's tiles to the right side and add the same number, size, and color tiles to the left side.



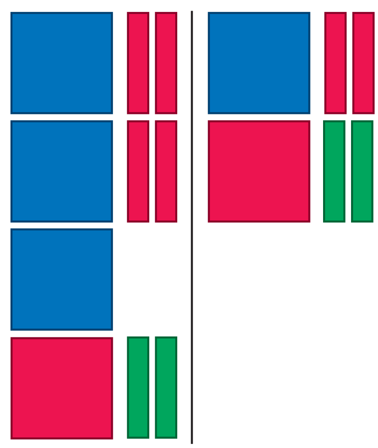
Pair each yellow tile with a red tile on the right side and remove the zero pairs. This should empty the right side. The remaining tiles model the equation in standard form.



Example 2: $3x^2 - 4x = x^2 - 2x$
Answer: $2x^2 - 2x = 0$; quadratic
Model the equation by placing three x^2 tiles (blue) and four red (negative) x tiles on the left side of the mat. Place one blue (positive) x^2 tile and two red (negative) x tiles on the right side.



To the right side of the mat, add one red x^2 tile and two green x tiles. Add the same number, color, and size tiles to the right.



Pair the positive and negative tiles of the same size on the right and remove each zero pair. The right side should become empty. The remaining tiles model the equation in standard form.