12.7

# teacher notes

## Objective

- Divide polynomials by factoring.

## Vocabulary

Factoring (Lesson 12-1)

Commutative Property of Addition (Lesson 2-3)

## Prerequisites

Factoring polynomials completely

Canceling common factors to simplify fractions

Indicating a division as a fraction

## Get Started

- Write the division 105 ÷ 21 on the board. Instruct the students to find the quotient.
- Ask the students if there is another way to write 105 ÷ 21. The division can be written as the fraction $\frac{105}{21}$.
- Instruct the students to find the prime factorization of 105 and 21 and to rewrite the fraction as $\frac{3 \cdot 5 \cdot 7}{3 \cdot 7}$.
- Discuss the methods that can used to simplify this fraction. Ask if both the numerator and the denominator can be divided by three, and how that division can be shown. The division can be shown by crossing out the three in each place and putting the result of that division above the crossed out factor. Remind the students that this procedure is called cancellation. Repeat for the common factor of seven.

$$\frac{3 \cdot 5 \cdot 7}{3 \cdot 7} = \frac{1 \cdot 5 \cdot 1}{1 \cdot 1} = \frac{5}{1} = 5.$$

- Say, "When the numerator and denominator are written in their prime factorizations, common factors can be cancelled, each one becoming a factor of one. That makes it easier to find the quotient. In this case, the quantity three times five times seven, divided by the quantity three times seven, is equal to five divided by one, or five."
- Say, "Today, we'll see that the same strategy can be used to divide polynomials. When the expressions in the numerator and denominator are factored, factors common to the numerator and denominator can be canceled out, making it easier to find the quotient."

# Section 1

## Expand Their Horizons

In this lesson, the quotient of two polynomials will be found by writing the division problem as a fraction, completely factoring numerator and denominator and then canceling common factors. The divisions found in this section are limited to those in which the algebraic factors (those containing variables) in the denominator are eliminated by cancellation, making the denominator a constant. When the denominator is one, the quotient is equivalent to the expression in the numerator.

Students have some experience already with dividing polynomials. In Lesson 11-6, they saw how to divide a polynomial by a monomial by writing the quotient of a polynomial and a monomial as the sum of the quotients of each term and the monomial. For example,

$$\frac{2x+6}{2} = \frac{2x}{2} + \frac{6}{2} = x + 3.$$

In Lesson 11-7, they saw how to divide a polynomial by another polynomial using long division. For example,

$$\begin{array}{r} 3x - 1 \\ 2x + 3\overline{)\ 6x^2 + 7x - 3} \\ -6x^2 - 9x \\ \hline -2x - 3 \\ +\ 2x + 3 \\ \hline 0 \end{array}$$

In this lesson, the strategy for dividing polynomials is to express the given problem as a fraction, factor the numerator and denominator, and cancel common factors. The resulting fraction is rewritten showing the results of the cancellations and eliminating any unnecessary 1's. This fraction should be in simplest form, i.e., the numerator and denominator have no common factors other than one.

It is important to note that the methods used in this lesson result in the same quotients found using the methods of Lessons 11-6 and 11-7. For example, $\frac{2x+6}{2} = \frac{2x}{2} + \frac{6}{2} = x + 3$ and $\frac{2x+6}{2} = \frac{2(x+3)}{2} = x + 3$. Likewise, the answer shown in the long division above is the same as the answer resulting from the method in this lesson.

$$\frac{6x^2+7x-3}{2x+3} = \frac{(2x+3)(3x-1)}{(2x+3)} = \frac{1 \cdot (3x-1)}{1} = 3x - 1$$

Dividing polynomials by factoring is an efficient method when the numerator and denominator are easily factored. In order for the method to work, both expressions should be factored completely.

It may be necessary to remind the class that the topic of this lesson is *dividing* polynomials. Since the direction line in the materials reads "Simplify:", students may forget that they are *dividing* polynomials. Help remind them by occasionally referring to the numerator as the *dividend*, the denominator as the *divisor*, and the simplified expression as the *quotient*. It is important that students see the relationship between the two tasks.

In the problems of Section I, the denominator is a prime polynomial and the numerator can be factored. Since the object of factoring the numerator is to create a factor in the numerator that will cancel with the expression in the denominator, the expression in the denominator can be used as a "hint" as to how the numerator will factor. For example, in the expression $\frac{6x^2 - 5x - 4}{2x + 1}$, suggest that students attempt to factor the numerator so that one of the factors is $2x + 1$. Their factoring skills should be strong enough to realize that the second factor must be $3x - 4$ in order for the product of $2x + 1$ and $3x - 4$ to be $6x^2 - 5x - 4$. Encourage students to enclose numerators or denominators that will not factor in parentheses. The parentheses can act as a visual reminder to cancel the entire expression. This is suggested, and done in the DVD lesson.

**1** **Simplify: $\frac{b^2 - 4}{b + 2}$.** The expression $b^2 - 4$ is the difference of two squares and factors as $(b + 2)(b - 2)$. The factor $b + 2$ is common to the numerator and denominator and can be canceled. The quotient of $b^2 - 4$ and $b + 2$ is $b - 2$. $\frac{b^2 - 4}{b + 2} = \frac{(b + 2)(b - 2)}{(b + 2)} = b - 2$.

**2** **Simplify: $\frac{m^2 + 8m + 12}{m + 2}$.** The numerator is the product of $(m + 2)$ and $(m + 6)$. The common factor $m + 2$ cancels, making the quotient $m + 6$.

$$\frac{m^2 + 8m + 12}{m + 2} = \frac{(m + 2)(m + 6)}{(m + 2)} = m + 6$$

### Common Error Alert

Students may be overconfident in their abilities to factor expressions. Warn students that they *must* check the factorization to be sure it is correct before proceeding to canceling the common factors.

## Additional Examples

**1. Simplify: $\frac{8r^4 - 10r^2}{4r^2 - 5}$.**

Factor the numerator by removing the common factor $2r^2$.

$$\frac{8r^4 - 10r^2}{4r^2 - 5} = \frac{2r^2(4r^2 - 5)}{(4r^2 - 5)}$$
$$= 2r^2$$

**2. Simplify: $\frac{8x^2 + 14x - 15}{4x - 3}$.**

Factor the numerator by grouping, $(8x^2 + 20x) - (6x + 15) =$ $4x(2x + 5) - 3(2x + 5)$

$$\frac{8x^2 + 14x - 15}{4x - 3} = \frac{(4x - 3)(2x + 5)}{(4x - 3)}$$
$$= 2x + 5$$

# Section 2

## Expand Their Horizons

In Section 2, both the numerator and the denominator will be composite polynomials. Each expression must be factored before cancellation of common factors can occur.

In this lesson, the degree of the expression in the denominator is always less than that of the numerator. This situation often makes it the easier of the two expressions to factor, since it sometimes involves fewer factorizations to get to the prime factorization. Consider suggesting that students factor the denominator first in these problems. Also, the factorization of the denominator may provide a hint as to the more complicated factorization of the numerator. Be careful not to make this a rule, as this may not be a useful technique in future study when factorization is used to simplify algebraic fractions that may be more complicated.

**3** **Simplify: $\frac{30g^2 - 57g - 45}{6 + 10g}$.** Remove the common factor two from the terms of the denominator to get $2(3 + 5g)$. Remove the common factor three from the terms of the numerator to get $3(10g^2 - 19g - 15)$. Factor the trinomial and write the numerator as $3(5g + 3)(2g - 5)$. Since $5g + 3 = 3 + 5g$ by the Commutative Property of Addition, the expression can be cancelled from the numerator and denominator. The resulting expression can be written either as the rational expression $\frac{3(2g-5)}{2}$, as the product $\frac{3}{2}(2g - 5)$, or (using the Distributive Property) as the polynomial $3g - \frac{15}{2}$.

## Look Beyond

In this lesson, the quotient of two expressions was shown in fractional form. In Lesson 15-2, an expression written this way will be called a *rational expression*. In that lesson, students will simplify rational expressions as fractions rather than as indicated divisions. They will write rational expressions in simplest form by finding the prime factorizations of the numerator and denominator, canceling the common factors, and writing the simplified expressions for numerator and denominator. Unlike the fractions in this lesson, in which the denominator simplified to a constant or a simple binomial, in Lesson 15-2, the fractions will simplify to fractions having denominators that may include non-constant monomials and polynomials and that may appear to be larger than the numerator.

## Additional Examples

**1. Simplify: $\frac{-s^2 - 5s - 6}{-7s - 14}$.**

First remove the common factor –1 from the expressions in the numerator and denominator.

Then, remove the common factor of seven from $7s + 14$ in the denominator, and factor the trinomial in the numerator.

$$\frac{-s^2 - 5s - 6}{-7s - 14} = \frac{-1(s^2 + 5s + 6)}{-1(7s + 14)}$$
$$= \frac{-1(s + 2)(s + 3)}{-1(7)(s + 2)}$$
$$= \frac{s + 3}{7}$$

**2. Simplify: $\frac{3b^2 - 27}{5b - 15}$.**

Remove the common factors (five from the denominator and three from the numerator). Continue to factor the factor $b^2 - 9$ as the difference of two squares.

$$\frac{3b^2 - 27}{5b - 15} = \frac{3(b^2 - 9)}{5(b - 3)}$$
$$= \frac{3(b + 3)(b - 3)}{5(b - 3)}$$
$$= \frac{3(b + 3)}{5}$$