12.3 teacher notes

Objectives

- Factor the difference of two squares.
- Recognize the first 15 perfect squares.
- Recognize the sum of two squares is not factorable.

Vocabulary

Conjugates (Lesson 11-5)
Perfect square number
Factoring (Lesson 12-1)
FOIL Method (Lesson 11-4)

Prerequisites

Applying the *rules of exponents*

Multiplying binomials using the FOIL Method

Finding the area of a rectangle

Get Started

- Write the expression $x^2 - 9$ on the board and ask the how to factor it. If students suggest $(x - 3)^2$, show them that this expression is equal to $x^2 - 6x + 9$, not $x^2 - 9$.
- Next, write, below the expression $x^2 - 9$, the expression $x^2 - 3x + 3x - 9$ on the board. Ask the class to study the new expression and guide them to see that the two expressions are equivalent.
- Ask, "Now can you factor this second expression?" If necessary, remind the class of the technique of factoring by grouping. The common factor x can be removed from the first two terms; the common factor three can be removed from the second two terms. So, the expression can be factored in the following way: $x^2 - 3x + 3x - 9 = x(x - 3) + 3(x - 3) = (x - 3)(x + 3)$.
- Say, "The expression $x^2 - 9$ is a special form. It is a *difference of two squares*, or a perfect square minus another perfect square. x^2 is the square of x, and nine is the square of three. In today's lesson, we will learn a rule for factoring the difference of two squares."

- After completing the DVD lesson, return to the expression $x^2 - 9$ and confirm that the rule $a^2 - b^2 = (a + b)(a - b)$ can be used to factor the expression. Verify that the factorization from this activity and the factorization found using the rule are the same.

Section 1

Expand Their Horizons

In this lesson, students will learn the rule for factoring the difference of two squares. For any expressions a and b (where a and b may be monomials or any other kind of algebraic expressions), $a^2 - b^2 = (a + b)(a - b)$. The binomials $a + b$ and $a - b$ are called conjugates. Notice that the Commutative Property of Multiplication allows the rule to be written in the alternate form $a^2 - b^2 = (a - b)(a + b)$.

There are several ways to demonstrate the rule $a^2 - b^2 = (a + b)(a - b)$.. The "Getting Started" activity demonstrates the rule algebraically for the polynomial $x^2 - 9$; the manipulatives section of the DVD uses algebra tiles to demonstrate other examples.

The lesson begins with a geometric interpretation of the rule. If time permits, students should repeat the activity on their own. Provide each student with large-grid graph paper (grid on both sides) and scissors. Ask them to draw a large square and a smaller square inside the larger square so that the squares have a common vertex (Figure 1). Tell them to count blocks on the grid to determine the lengths of the sides of their squares, assigning the variable a to the side of the large square and the variable b to the side of the smaller square. (For example, one student may have $a = 7$ and $b = 3$, and another student may have $a = 9$ and $b = 6$.) Next, have them cut out the smaller square, leaving a six-sided figure with area $a^2 - b^2$ (Figure 2). They can count squares on the graph paper to confirm that the area of the figure is $a^2 - b^2$. Next, students should cut the figure along the line shown (Figure 3) and then, turn over and rotate one of the pieces to form a rectangle (Figure 4). Using his or her own values of a and b, students should confirm the rectangle has width $a - b$ and length $a + b$.

Figure 1

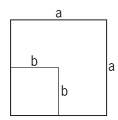


Figure 2

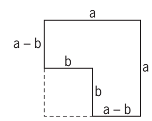


Figure 3

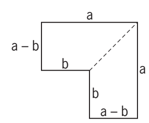
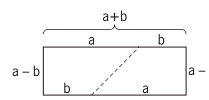


Figure 4



In order to decide whether a binomial is the difference of two squares, it is important to be able to recognize perfect square numbers. Students should memorize the squares of the numbers one through 15.

In one example, the class is asked to factor the expression $w^2 + 36$. This expression is the sum of two squares, not the difference, and therefore, cannot be factored using the rule learned in this lesson. There is no rule for factoring the sum of squares. However, a sum of squares may be factorable if its terms have a common factor. For example, $16x^2 + 4 = 4(4x^2 + 1)$. Note: the example listed is not a sum of squares.

Common Error Alert

Some students may factor the difference of two squares and get the square of a binomial as a result. For example, they may factor $b^2 - 100$ and get $(b - 10)^2$ as a result. Remind students to check their factorizations by multiplying. Since $(b - 10)^2 = (b - 10)(b - 10) = b^2 - 20b + 100$, $b^2 - 100 \neq (b - 10)^2$.

1 **Factor, if possible: $b^2 - 100$.** Since b^2 is the square of b and 100 is the square of 10, the expression $b^2 - 100$ is the difference of two squares and can be factored: $b^2 - 100 = (b + 10)(b - 10)$.

Common Error Alert

Students may have an answer correct and think it is incorrect if the binomial factors do not appear in the same order as the order given in the DVD. Remind them that the Commutative Property of Multiplication allows the order of a factorization to be reversed. For example, $1 - z^2$ can be written as either $(1 + z)(1 - z)$ or $(1 - z)(1 + z)$.

2 **Factor, if possible: $1 - z^2$.** Students may need some additional explanation for this expression because the first term is a number, and in all previous expressions, the first term was a variable. The binomial $1 - z^2$ is the difference of the square of one and the square of z: $1 - z^2 = 1^2 - z^2 = (1 + z)(1 - z)$.

Common Error Alert

Students may attempt to apply the Commutative Property of Addition to the terms in a difference binomial. For example, they may *incorrectly* think that $(1 - z)$ can be replaced by $(z - 1)$. Emphasize that the Commutative Property *does not apply to subtraction*. For example, $(1 + z)(1 - z) \neq (z + 1)(z - 1)$, because $(1 - z) \neq (z - 1)$.

Additional Examples

1. Factor, if possible: $n^2 - 400$.

Write each term as the square of a monomial. Then, use the bases n and 20 in the factorization.

$$n^2 - 400 = (n)^2 - (20)^2$$
$$= (n + 20)(n - 20)$$

2. Factor, if possible: $\frac{1}{4} - k^2$.

Use the fact that $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

$$\frac{1}{4} - k^2 = \left(\frac{1}{2}\right)^2 - k^2$$
$$= \left(\frac{1}{2} + k\right)\left(\frac{1}{2} - k\right)$$

Section 2

Expand Their Horizons

In Section 2, two additional cases of factoring the difference of two squares will be studied.

The first case addresses the problem of factoring the difference of two squares with a leading coefficient not equal to one. To factor an expression like $4x^2 - 25$, students must be able to identify $4x^2$ as the square of the monomial $2x$. Review the following rules of exponents with the class: To raise a power to a power, multiply the exponents; to raise a monomial to a power, raise each factor of the monomial to the power. So, $(2x)^2 = 2^2 \cdot x^2 = 4x^2$. Let students practice rewriting expressions that are squares of monomials, such as: $16x^2 = (4x)^2$, $64b^4 = (8b^2)^2$, $121m^6 = (11m^3)^2$.

The second case of factoring the difference of two squares addresses the instance in which the factoring rule must be applied more than once to completely factor an expression. This is the case with the expression $x^4 - 16$. The first application of the factoring rule results in

$(x^2 + 4)(x^2 - 4)$, but the factor $x^2 - 4$ itself can be factored; therefore, another factoring step is needed. The complete factorization of $x^4 - 16$ is $x^4 - 16 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x + 2)(x - 2)$. Emphasize that the direction "Factor" means "Factor completely."

3 **Factor, if possible: $100h^2 - 49$.** For expressions like this one, in which one of the terms has both a coefficient and a variable, encourage students to rewrite the expression first to show the squares explicitly: $100h^2 - 49 = (10h)^2 - (7)^2 = (10h + 7)(10h - 7)$.

Look Beyond

The rule for factoring the difference of squares, $a^2 - b^2 = (a + b)(a - b)$, is shown in this lesson only for expressions in which a and b are monomials. However, a and b can be expressions that are not monomials. They can be polynomials of more than one term or trigonometric expressions, just to name two types. An example of each type is shown below.

$(x + y)^2 - 4^2 = [(x + y) + 4][(x + y) - 4)]$
$= (x + y + 4)(x + y - 4)$

$\sin^2 x - \cos^2 x = (\sin x)^2 - (\cos x)^2 =$
$(\sin x + \cos x)(\sin x - \cos x)$

Note: $\sin^2 x$ means $(\sin x)^2$.

Additional Examples

1. Factor, if possible: $100 - 121d^6$.

One hundred is the square of 10; $121d^6$ is the square of $11d^3$.

$$100 - 121d^6 = (10)^2 - (11d^3)^2$$
$$= (10 + 11d^3)(10 - 11d^3)$$

2. Factor, if possible: $36m^4 + 100$.

This binomial is the sum of squares. The addends have a greatest common factor of four. Factor by removing the GCF.

$36m^4 + 100 = 4(9m^4 + 25)$