11.6 teacher notes

Objectives

- Divide a monomial by a monomial.
- Divide a binomial or a trinomial by a monomial.

Vocabulary

Polynomial (Lesson 2-1)
Binomial (Lesson 2-1)
Trinomial (Lesson 2-1)
Base (Lesson 1-4)
Exponent (Lesson 1-4)
Division rule for exponents (Lesson 11-1)

Prerequisites

Using the *division rule for exponents*

Performing operations with integers

Reducing fractions by canceling common factors

Get Started

- Divide the class into pairs for this activity. Say, "Today, I am going to ask you to help determine whether a certain algebraic property is true. One way to gain evidence a property is true is to show it works for a certain set of real numbers. For example, you can show the Commutative Property of Addition, $a + b = b + a$, is true by substituting three for a and five for b to get $3 + 5 = 5 + 3$, or $8 = 8$."
- Write the equations $a + b = b + a$; $3 + 5 = 5 + 3$; and $8 = 8$ on the board in successive rows with equal signs aligned. Write "TRUE" under the last equation. Point out in the first row, the property was stated; in the second row, real values were substituted; and in the last row, the expressions were simplified to show they were true.
- Say, "Now, let's look at the equation $\frac{a+b+c}{d} = \frac{a}{d} + \frac{b}{d} + \frac{c}{d}$." Write the equation on the board. "This equation shows the quotient of a sum and an expression 'broken up' into the sum of quotients terms; each of which shows one addend from the numerator divided by the denominator." Show the class how each term of the numerator is used to make a fraction with the denominator d.

- Say, "Working with your partner, substitute real numbers for a, b, c, and d and show this property is true. Use the same format I used when I showed you the Commutative Property of Addition was true when $a = 3$ and $b = 5$."
- Allow students time to work. If necessary, guide them to choose convenient values for the variables so that a, b and c, are multiples of d.
- Say, "In today's lesson, we will use this property to divide polynomials by monomials."

Section 1

Expand Their Horizons

In this lesson, students will be shown how to divide one monomial by another. Then, they will be shown how to divide a polynomial (of two or three terms) by a monomial. To divide a polynomial by a monomial, each term of the polynomial is divided by the monomial, and the individual quotients are added.

As a review of some vocabulary, in the exponential expression a^n, a is the base, and n is the exponent. In a monomial, the coefficient is the numerical factor; in $-2a^2b^4$, the coefficient is -2. To divide a monomial by a monomial, first simplify the coefficients by canceling common factors. Then, use the *division rule for exponents* to simplify powers with like bases. If the coefficient in the numerator is not a multiple of the coefficient in the denominator, the final quotient can be written in fractional form. For example, $\frac{24x^4y}{15x^2}$ is simplified to $\frac{8x^2y}{5}$. This expression can be written in monomial form as $\frac{8}{5}x^2y$. It is important that students see $\frac{8x^2y}{5}$ and $\frac{8}{5}x^2y$ are equivalent expressions, with the former being a quotient and the latter a monomial.

1 **Simplify: $\frac{-b^5d^2}{b^3d}$.** In this quotient, the coefficient of the numerator is negative one and the coefficient of the denominator is one. It may help some students to write these coefficients into the problem. The coefficient of the quotient is $\frac{-1}{1}$ or -1. The factors b^5 and b^3 have like bases and can be simplified using the *division rule for exponents* as $\frac{b^5}{b^3} = b^{5-3} = b^2$. The factors d^2 and d have like bases and can be simplified as $\frac{d^2}{d^1} = d^{2-1} = d^1 = d$. Recall d equals d^1. The product of the individual quotients is $-1 \cdot b^2 \cdot d = -b^2d$.

2 **Simplify: $\frac{35s^4t^4}{14st^{-2}}$.** This expression contains a negative exponent. To simplify the t terms, apply the *division rule for exponents*. So, $\frac{t^4}{t^{-2}} = t^{4-(-2)} = t^{4+2} = t^6$. Another method for simplifying the t terms is to apply the definition of negative exponents first; then, use the *multiplication rule for exponents*. Remind the class $\frac{1}{a^{-n}} = a^n$. So, $\frac{t^4}{t^{-2}} = t^4 \cdot t^2 = t^{4+2} = t^6$. For the s terms, $\frac{s^4}{s}$ equals s^3. The coefficients 35 and 14 simplify to $\frac{5}{2}$. This gives $\frac{5}{2}s^3t^6$.

Additional Examples

1. Simplify: $\frac{-18x^3y^5}{3x^3y}$.

The factors x^3 and x^3 have a quotient of one. A power with base x will not appear in the quotient.

$$\begin{aligned}\frac{-18x^3y^5}{3x^3y} &= \frac{-18}{3} \cdot \frac{x^3}{x^3} \cdot \frac{y^5}{y}\\ &= -6 \cdot x^{3-3} \cdot y^{5-1}\\ &= -6 \cdot x^0 \cdot y^4\\ &= -6 \cdot 1 \cdot y^4\\ &= -6y^4\end{aligned}$$

2. Simplify: $\frac{4r^{-2}s^2}{12r^{-4}s}$.

Apply the *division rule for exponents*, so the power of r is $-2 - (-4)$ or apply the definition of negative exponents to each negative exponent to rewrite the expression as $\frac{4r^4s^2}{12r^2s}$.

$$\begin{aligned}\frac{4r^{-2}s^2}{12r^{-4}s} &= \frac{4r^4s^2}{12r^2s}\\ &= \frac{4}{12} \cdot \frac{r^4}{r^2} \cdot \frac{s^2}{s}\\ &= \frac{1}{3} \cdot r^{4-2} \cdot s^{2-1}\\ &= \frac{1}{3}r^2s\end{aligned}$$

Section 2

Expand Their Horizons

In Section 2, binomials and trinomials will be divided by monomials. The technique shown in this lesson can be used to divide any polynomial by a monomial. To find the quotient of a binomial (trinomial) and a monomial, create two (three) individual quotients, each one showing the quotient of one term of the numerator and the denominator. Simplify each fraction; then, add to find the final quotient.

Common Error Alert

When simplifying the expression $\frac{6b + 3}{3}$, students may want to cancel the 3's. Remind students only *factors* that are common to the numerator and denominator can be canceled. *Terms* that are common cannot be reduced.

To make this point clear, ask students to evaluate $\frac{6 + 4}{2}$ (it is equivalent to five). Then, ask them to cancel common terms to get $\frac{6 + \cancel{4}^{2}}{\cancel{2}_{1}} = \frac{6 + 2}{1} = 8$. Contrast with the example $\frac{6 \cdot 4}{2}$ (which is equivalent to 12). This expression can be legitimately simplified by canceling common *factors* to get $\frac{6 \cdot \cancel{4}^{2}}{\cancel{2}_{1}} = \frac{6 \cdot 2}{1} = 12$. First factor the two out of both terms and then cancel.

3 **Simplify:** $\frac{16c^2d - 8cd^2}{4cd}$**.** Write the quotient as the sum of two quotients. Simplify each quotient and then, write their sum. This expression equals $\frac{16c^2d}{4cd} - \frac{8cd^2}{4cd}$ or $4c - 2d$. Factor out $4cd$ making the numerator $4cd(4c - 2d)$ and then, cancel the $4cd$.

Additional Examples

1. Simplify: $\dfrac{3bc^5 + 12b^4c^3}{6bc^2}$.

Write as two fractions, simplify each, and then write the sum.

$$\begin{aligned}\frac{3bc^5 + 12b^4c^3}{6bc^2} &= \frac{3bc^5}{6bc^2} + \frac{12b^4c^3}{6bc^2}\\ &= \frac{1}{2}b^{1-1}c^{5-2} + 2b^{4-1}c^{3-2}\\ &= \frac{1}{2}c^3 + 2b^3c\end{aligned}$$

2. Simplify: $\dfrac{4m^3n + 8m^2n^2 - 4mn^3}{2mn}$.

Write as three fractions, simplify each, and then write the sum.

$$\begin{aligned}\frac{4m^3n + 8m^2n^2 - 4mn^3}{2mn} &= \frac{4m^3n}{2mn} + \frac{8m^2n^2}{2mn} - \frac{4mn^3}{2mn}\\ &= 2m^{3-1}n^{1-1} + 4m^{2-1}n^{2-1} - 2m^{1-1}n^{3-1}\\ &= 2m^2 + 4mn - 2n^2\end{aligned}$$

Look Beyond

In future lessons, students will be asked to remove a common factor from a polynomial in order to find its prime factorization. This task requires that multiplication be "undone," so division is necessary. For example, when the common factor ab is removed from the expression $a^3b - ab^2$, the result is $ab(a^2 - b)$. To remove a common factor from the terms of a polynomial, each term in $a^3b - ab^2$ is divided by the common factor ab, and the quotients form the second factor in the factorization $ab(a^2 - b)$.