11.5 teacher notes

Objectives

- Use the Product of Conjugates pattern to find the product of two binomials.
- Use the Square of a Sum and Square of a Difference patterns to find the product of two binomials.
- Use horizontal and vertical methods to multiply polynomials using the Distributive Property.

Vocabulary

FOIL Method (Lesson 11-4)
Special products
Product of Conjugates
Conjugates
Square of a Sum
Square of a Difference
Difference of two squares

Prerequisites

Multiplying a monomial by a binomial

Multiplying a binomial by a binomial using the FOIL Method

Combining like terms

Applying the *rules of exponents*

Get Started

- Write the expression $(a + b)^2$ on the board and ask students to guess what the expanded form of the expression would be. Students may incorrectly say $(a + b)^2 = a^2 + b^2$.
- Say, "Let's test the idea that $(a + b)^2 = a^2 + b^2$ by substituting real values for a and b." Write the expression $(3 + 4)^2$ on the board and ask students to evaluate the expression using the order of operations. The correct answer is 49.
- Say, "We know that $(3 + 4)^2 = 49$. Can we get the same result using the idea that $(a + b)^2 = a^2 + b^2$? Is it true that $(3 + 4)^2$ is equal to $3^2 + 4^2$? Give students a moment to determine $3^2 + 4^2 = 9 + 16 = 25$, not 49.
- If time permits, repeat the activity to show $(a - b)^2 \neq a^2 - b^2$, by demonstrating that $(10 - 7)^2 \neq 10^2 - 7^2$.

- Say, "Squaring a binomial means finding the product of two binomials, which means we should use the FOIL Method. In this lesson, we will use the FOIL Method to develop rules for finding the products of binomial factors that take certain forms. We call these forms *special products.* One such special product is the square of a sum, which we saw when finding the square of the sum of a and b. The rule we develop will enable us to find the square of a binomial like $x + 5$ without having to use the FOIL Method."
- If time permits an additional activity, say, "Another special product is the square of a difference. So, we'll also develop a rule for finding the square of a binomial like $x - 5$ without having to use the FOIL Method.

Section 1

Expand Their Horizons

In this lesson, students will use the FOIL Method of multiplying binomials to develop "shortcut" rules for finding the square of a sum, the square of a difference, and the product of conjugates. They are also shown how to multiply a binomial by a polynomial, using both horizontal and vertical formats.

Understanding the FOIL Method is essential to developing the special-product rules in this lesson. Before starting the lesson, students should review how to find the product of two binomials using the FOIL Method. Write the example $(x + 4)(x - 3)$ on the board and ask students to multiply the binomials. Remind them, when finding the product of two linear binomials written in decreasing degree, the Outer and Inner products are like terms and will always combine.

The first special case presented is the product of conjugates. *Conjugates* are expressions of the form $a + b$ and $a - b$, showing the sum and difference of terms a and b. When conjugates are multiplied using the FOIL Method, the Outer and Inner terms are always opposites and have a sum of zero when combined. So, $(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$. Be sure to identify the expression $a^2 - b^2$ as "the difference of two squares," because it will be important for students to recognize this special kind of binomial when factoring in Module 12.

1 **Simplify: $(5 + r)(5 - r)$.** Reassure students the expressions $5 + r$ and $5 - r$ are conjugates, despite not being written in order of decreasing degree. Because the expressions show the sum and difference of identical terms, they are conjugates, and the rule can be applied. $(5 + r)(5 - r) = 5^2 - r^2 = 25 - r^2$.

The square of any binomial can be found using the special product rules $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$. Students may need repeated exposure to the development of the rules to best understand the source of the middle term of the product.

2 **Simplify: $(5y + 1)^2$.** In this expression, the first term of the binomial is $5y$, and the second is one. The first term of the product is $(5y)^2 = 25y^2$. The middle term is $(2)(5y)(1) = 10y$. The last term is $(1)^2 = 1$. So, $(5y + 1)^2 = 25y^2 + 10y + 1$.

When students find the square of a binomial, they may state the product as the sum (or difference) of the squares of the terms.

For example, they might say $(5y + 1)^2 = 25y^2 + 1$. Refer back to the "Get Started" activity. Use a real number example to show $(a + b)^2 \neq a^2 + b^2$. Using the FOIL Method will help students who have difficulty applying the shortcut rules.

3 **Simplify: $(m - 4n)^2$.** This expression shows the Square of a Difference, according to the rule $(a - b)^2 = a^2 - 2ab + b^2$. Let $a = m$ and $b = 4n$; then, $(m - 4n)^2 = m^2 - 2(4mn) + (4n)^2 = m^2 - 8mn + 16n^2$.

When finding the Square of a Difference, students should write the correct signs in the product. For $(m - 4n)^2$, they may write the product as $m^2 - 8mn - 16n^2$. Students should refer to the stated rule to check the signs. Remind them the middle term is twice $-4mn$ (the product of m and $-4n$), so it is negative. The term $16n^2$ is positive, because it is the product of $-4n$ and $-4n$. Encourage students to show all the steps in the calculation. For example, they should write $(m - 4n)^2 = m^2 - 2(4mn) + (4n)^2 = m^2 - 8mn + 16n^2$. If some students have difficulty remembering the rules for special products, remind them that all the special products can be found using the FOIL Method.

The last portion of the lesson shows how to multiply a binomial by a trinomial. Finding the product of two polynomials requires repeated applications of the Distributive Property. To find the product of the binomial $x + 5$ and the trinomial $x^2 - 5x + 1$, multiply each term of the binomial (x and 5) by each term of the trinomial x^2, $-5x$, and 1. Then, write the sum of those six products and simplify by combining like terms.

Another way to perform the multiplication $(x + 5)(x^2 - 5x + 1)$ is to distribute the binomial $x + 5$ to each term in the trinomial. This step yields the expression $(x + 5)(x^2) - (x + 5)(5x) + (x + 5)(1)$. Ultimately, this method will result in the same six terms as the method described in the previous paragraph. It might be a useful exercise for students to show that both methods result in the same final product.

A third way to find the product of two polynomials is using the vertical format described in the lesson. Again, this method results in six individual products (identical to those found in the other two methods presented) that are added to find the final product. Students are already familiar with the algorithm for vertical multiplication because they have almost certainly used this method for multiplying multi-digit numbers. When multiplying numbers, each digit of the bottom factor is multiplied by each digit of the top factor, and the resulting products are written in the appropriate decimal places on successive lines of the workspace. When each digit has been multiplied, the products in the workspace are added to find the final answer.

When polynomials are multiplied in this way, like terms are aligned vertically (just as numerals must be aligned according to place value when multiplying real numbers). To multiply, the last term of the bottom factor is multiplied by each term of the top factor. That product is written on the first line below the bar. Working from right to left, the process is repeated with each term of the bottom factor, with the products being written on successive lines. If done correctly, like terms should be aligned, and combining like terms will require only adding coefficients.

Common Error Alert

Students sometimes become confused about which terms to multiply when using a vertical format to multiply polynomials. They may lose track of their work. To help avoid this, teach them to circle the term being distributed, including the preceding addition or subtraction sign. Working from right to left in the bottom polynomial, after a term has been distributed, they can cross it out and circle the next term. Each time a new term is circled, they move on to a new line to record the products.

Additional Examples

1. Multiply using the rule for the Square of a Difference: $(2x - 5)^2$.

$$\begin{aligned}(2x - 5)^2 &= (2x)^2 + (2)(2x)(-5) + (5)^2\\ &= 4x^2 - 20x + 25\end{aligned}$$

2. Multiply using a vertical format: $(2x + 3)(2x^3 - x + 1)$.

Write the polynomial with more terms on the top.

$$\begin{array}{rrrrr} & 2x^3 & & -\ x & +\ 1\\ \times & & & 2x & +\ 3\\ \hline & 6x^3 & & -3x & +\ 3\\ +\ 4x^4 & & -\ 2x^2 & +\ 2x & \\ \hline 4x^4 & +\ 6x^3 & -\ 2x^2 & -\ x & +\ 3\end{array}$$

Connections

The rules for finding the Square of a Sum and the Square of a Difference can be used in mental math. For example, to find the square of 29, use the following calculation:

$$\begin{aligned}29^2 &= (20 + 9)^2\\ &= (20)^2 + (2)(20)(9) + (9)^2\\ &= 400 + 360 + 81\\ &= 841\end{aligned}$$

In addition, the techniques for multiplying polynomials can be used to find the product of any two numbers. For example, $52 \cdot 213$ can be found by the following two methods:

$$\begin{aligned}(50 + 2)(200 + 10 + 3) &= 50 \cdot 200 + 50 \cdot 10 + 50 \cdot 3 + 2 \cdot 200 + 2 \cdot 10 + 2 \cdot 3\\ &= 10{,}000 + 500 + 150 + 400 + 20 + 6\\ &= 11{,}076\end{aligned}$$

or

$$\begin{array}{rrrr} & 200 & +\ 10 & +\ 3\\ \times & & 50 & +\ 2\\ \hline & 400 & +\ 20 & +\ 6\\ +\ 10{,}000 & +\ 500 & +\ 150 & \\ \hline 10{,}000 & +\ 900 & +\ 170 & +\ 6\\ & & & 11{,}076\end{array}$$

Look Beyond

In the next unit, students will learn how to factor polynomials. In factoring, a polynomial is written as the product of two or more other polynomials. For example, the binomial $x^2 - 9$ can be factored as the product $(x + 3)(x - 3)$. A thorough understanding of multiplication of polynomials, and especially of the FOIL Method, is essential to learning the factoring method students will use in the future.

Special products appear throughout the topics of algebra. For example, the standard form for the equation of a parabola is $y - k = a(x - h)^2$, which contains the square of a difference. Writing the equation as a polynomial function requires squaring the difference.