11.3 teacher notes

Objectives

- Add polynomials.
- Subtract polynomials.

Prerequisites

Adding and subtracting integers

Identifying like terms

Combining like terms

Applying rules of exponents

Using the Distributive Property of Multiplication over Addition

Vocabulary

Polynomial (Lesson 2-1)
Monomial (Lesson 2-1)
Binomial (Lesson 2-1)
Trinomial (Lesson 2-1)
Like terms (Lesson 2-4)
Distributive Property of Multiplication over Addition (Lesson 2-3)

Get Started

- Separate the class into two groups.
- Instruct each group to separate into at least three smaller groups according to hair color (for example, blond, brunette, and red hair).
- Tell the groups to write a polynomial that represents their division into the hair-color groups. For example, if one group is divided into five blonds, two brunettes, and three red-heads, they could write 5 BL + 2 BR + 3 BH as their representative polynomial.
- Write each group's polynomial on the board.
- Bring the class together as one large group and have the students separate according to hair color once again. Select a student to write a polynomial expression based on hair color for the entire class.
- Add the first group's polynomial to the second group's polynomial and demonstrate that the sum is equal to the "class polynomial." Discuss "like terms" while adding the polynomials. Students will learn more about this topic in today's lesson.

Section 1

Expand Their Horizons

In Section 1, students will add polynomials horizontally and vertically. Several definitions will be reviewed:

- A term is a number, a variable, or a product of numbers and variables.
- A monomial is an expression containing one term.
- A binomial is an expression containing two terms.
- A trinomial is an expression containing three terms.
- A polynomial is a monomial or the sum of monomials; it may contain any number of terms.

Common Error Alert

Students may mistakenly add the exponents when adding like terms. According to the rules of exponents, when multiplying like variables, the exponents need not be the same and are added: $x^2 \cdot x^3 = x^5$. However, when adding like variables, their corresponding exponents must be the same, and these exponents remain unchanged while the coefficients of the terms are added: $x^2 + x^2 = 2x^2$.

Adding polynomials involves combining like terms. Students should review several examples identifying like terms. Like terms can be constants or variables. To be like terms, like variables must be raised to like powers.

The terms 4 and 7 are like terms, while 4 and $7x$ are not, because one is a constant term and the other a variable term. The terms $9x$ and $5x$ are like terms, while $9x$ and $5y$ are not, because they have different variables. The terms $2x^2y^3$ and x^2y^3 are like terms, while $2x^2y^3$ and x^2y^3 are not, because the exponents of corresponding variables are not the same. The following are examples of adding like terms: $3 + 5 = 8$, $3x + 5x = 8x$, and $3x^2 + 5x^2 = 8x^2$.

When adding polynomials horizontally, the parentheses may be removed and like terms placed together to be combined. Consider the example $(3x^2 + 5x - 6) + (x^2 - 4x - 2)$. Removing the parentheses and placing like terms together yields $3x^2 + x^2 + 5x - 4x - 6 - 2$. To add like terms, add their coefficients. The sum in this example is $4x^2 + x - 8$.

Polynomials may also be added vertically. Consider $(2x^3 - 7x^2 + 3) + (-x^3 + 5x + 12)$. Remember to line up like terms and then add:

$$\begin{array}{rrrr} 2x^3 & - 7x^2 & & + 3 \\ + -x^3 & & + 5x & + 12 \\ \hline x^3 & - 7x^2 & + 5x & + 15 \end{array}$$

1 **Add: $(5r - 6) + (-5r + 6)$.** Adding the like terms $5r$ and $-5r$ yields zero. Adding the constant terms -6 and 6 also yields zero. Thus, the sum of the polynomials is zero.

2 **Add: $(3x^2 + 2xy - y^2) + (x^2 - xy + 2y^2)$.** Adding the coefficients of the x^2 terms, 3 and 1, results in $4x^2$. Adding the coefficients of the xy terms, 2 and -1, results in $1xy$ or xy. Adding the coefficients of the y^2 terms, -1 and 2, results in $1y^2$ or y^2. The sum is $4x^2 + xy + y^2$.

Additional Examples

1. Add vertically:
$(5x^2 + 3x - 1) + (x^2 + 4x) + (2x^2 - 7x - 6)$.

Removing the parentheses, then aligning and adding like terms yields:

$$\begin{array}{rrrr} & 5x^2 & + 3x & - 1 \\ & x^2 & + 4x & \\ + & 2x^2 & - 7x & - 6 \\ \hline & 8x^2 & & - 7 \end{array}$$

The sum of this expression is $8x^2 - 7$.

2. Add horizontally:
$\left(y^{\frac{2}{3}} + 5y^{\frac{1}{3}} - 4\right) + \left(3y^{\frac{2}{3}} - 6y^{\frac{1}{3}} + 1\right)$.

Although the exponents are fractions rather than whole numbers, like terms are still combined.

$y^{\frac{2}{3}} + 3y^{\frac{2}{3}} = 4y^{\frac{2}{3}}$

$5y^{\frac{1}{3}} - 6y^{\frac{1}{3}} = -y^{\frac{1}{3}}$

$-4 + 1 = -3$

The sum of this expression is $4y^{\frac{2}{3}} - y^{\frac{1}{3}} - 3$.

Section 2

Expand Their Horizons

In Section 2, students will subtract polynomials horizontally and vertically. As with adding polynomials, subtracting polynomials involves combining like terms. Consider the expression $(5y^2 + 2y - 7) - (y^2 - 4y + 3)$. Each pair of like terms is subtracted. For the squared terms, $5y^2 - y^2$ is $4y^2$. For the linear terms, $2y - (-4y)$ is $6y$. For the constant terms, $(-7) - 3$ is -10. Thus, the difference is $4y^2 + 6y - 10$. Subtraction is the same as adding the opposite. For ease of computation, the -1, which is understood to be outside the second left parenthesis, is distributed over that polynomial. That is, when the parentheses are removed, the operation is changed from subtraction to addition, and the signs of each term of the subtrahend (the second polynomial) are changed. In the example, $(5y^2 + 2y - 7) - (y^2 - 4y + 3)$ is equivalent to $5y^2 + 2y - 7 - y^2 + 4y - 3$. Like terms are added to get the same result: $4y^2 + 6y - 10$.

Although an answer in the form $7y^3 - y + 6y^2 - 4$ is mathematically correct, the preferred form is to list the variables in descending exponential order, with the constant term last (because it is the zero power of the variable). In fact, if other operations were to be performed on this polynomial, descending order might be required. Encourage students to place results in descending order: $7y^3 + 6y^2 - y - 4$.

The same technique for changing the operation to addition and for distributing negative one throughout the second polynomial can be used when subtracting polynomials vertically:

$$\begin{array}{l} (7x^3y + xy^2 - 9y^3) \\ \underline{-\ (4x^3y - x^2y - 6y^3)} \end{array} \text{ becomes}$$

$$\begin{array}{rrrrr} & 7x^3y & & + xy^2 & - 9y^3 \\ + & (-1)(4x^3y & - x^2y & & - 6y^3) \\ \hline \end{array} \text{ or}$$

$$\begin{array}{rrrrr} & 7x^3y & & + xy^2 & - 9y^3 \\ + & -4x^3y & + x^2y & & + 6y^3 \\ \hline & 3x^3y & + x^2y & + xy^2 & - 3y^3 \end{array}$$

3 **Subtract: $(y^2 + 2y + 8) - (y^2 - 2y + 4)$.** Distribute negative one over the second polynomial and remove all parentheses, to obtain the expression $y^2 + 2y + 8 - y^2 + 2y - 4$. Combine the terms y^2 and $-y^2$ to get zero. Combine the terms $2y$ and $2y$ to get $4y$ and combine the constant terms 8 and –4 to yield four. The difference of these polynomials is $4y + 4$.

Common Error Alert

Students should change the sign of all terms in the subtracted polynomial (the subtrahend), even those for which there is no like term with which to combine. This will ensure correct signs in the final result. Consider the subtraction example $(5y^3 - y - 3) - (-2y^3 - 6y^2 + 1)$. Rewriting the equation to remove the parentheses requires changing the operation to addition and reversing the signs of the second polynomial throughout. The expression becomes $5y^3 - y - 3 + 2y^3 + 6y^2 - 1$. Combining like terms yields $7y^3 + 6y^2 - y - 4$.

Look Beyond

The concept of adding like terms is widespread in science and mathematics classes. In geometry and trigonometry, angles are measured in degrees, minutes, and seconds and must be added by adding like parts: degrees to degrees, minutes to minutes, and seconds to seconds. Later in this course, rational expressions with "like denominators" will be added and subtracted and radical terms with "like radicands" and "like indices" will be combined.

Connections

Financial advisors, who need to find sums of money, must have like terms; dollars and yen cannot be added to yield an understandable total. Architects must add like measurements; feet must be added to feet and inches to inches. Pharmacists must also add like measurements; ounces and milligrams cannot be added. The need for recognizing like units and combining them can be found in many different professions.

Additional Examples

1. Subtract: $(2x^3 - 5x) - (x^2 - 7)$.

Rewrite the expression without the parentheses, being sure to add the opposite of the terms in the second polynomial. This yields $2x^3 - 5x - x^2 + 7$. Because there are no like terms to combine, merely place the terms in descending order for the final answer: $2x^3 - x^2 - 5x + 7$.

2. Subtract: $(5x^2 - 7) - (2x^2 - 7)$.

Rewrite the expression without the parentheses, being sure to add the opposite of the terms in the second polynomial. This yields $5x^2 - 7 - 2x^2 + 7$. Combine like terms to get $3x^2$.

Manipulatives

To demonstrate adding and subtracting polynomials with manipulatives, algebra tiles can be used. Small square tiles are used as ones tiles; that is, the small square tiles will represent the constant in the polynomial. These tiles have different colors on each side to represent positive and negative values. In this lesson, the positive side is represented with yellow and the negative side with red. Rectangular tiles are used as x tiles; that is, they represent the linear term (x term) in the polynomial. For the x tiles, green is the positive side, and red is the negative side. Finally, large square tiles—with sides of length equal to that of the longer side of the rectangular x tiles—are used as the x-squared tiles; that is, they represent the quadratic (x^2 term) in the polynomial. For these, blue is the positive side, and red is the negative side.

To add polynomials, represent each with tiles, using the red side whenever a negative coefficient or constant is shown. Like tiles (according to size) are grouped together. This is comparable to combining like terms. For those like tiles that have both positives and negatives represented, each pair (one positive and one negative) equals zero and should be discarded. The remaining tiles represent the result, where red tiles indicate a negative coefficient or constant.

Consider the example $(3x^2 + 2x - 5) + (x^2 - 7x - 1)$. The first polynomial is represented by three large square blue tiles (for $3x^2$), two rectangular green tiles (for $2x$), and five small square red tiles (for -5). The second polynomial is represented by one large square blue tile (for x^2, which is $1x^2$), seven rectangular red tiles (for $-7x$), and one small square red tile (for -1). Notice the negative terms are indicated by placing the red side of the tile up. Tiles are then grouped according to size: there are four large square blue tiles, two green and seven red rectangular tiles, and six small square red tiles. Within the group of rectangular tiles, the two green tiles can be paired with two red tiles and discarded since each pair represents zero. This leaves five red rectangular tiles. The solution is $4x^2 - 5x - 6$ as represented by four positive x^2 tiles, five negative x tiles, and six negative one tiles.

To subtract polynomials, represent each term with tiles, again using the red side whenever a negative coefficient or constant is shown. Therefore, each tile used to represent the subtrahend polynomial (the polynomial being subtracted, which is the second one) must be turned over to become its opposite. The negative tiles become positive and the positive tiles become negative. Then the tiles are grouped according to size, "0 pairs" are removed, and the result is computed.

Consider the example $(4n + 5) - (3n - 1)$. The first polynomial is represented by four rectangular green tiles (for $4n$) and five small square yellow tiles (for 5). The second is represented by three rectangular green tiles (for $3n$) and one small square red tile (for -1). The tiles in the second polynomial are turned over, to indicate addition of the opposite. Now the second polynomial is represented with three rectangular red tiles and one small square yellow tile. Grouping like sizes together, there are four green and three red rectangular tiles and six small square yellow tiles. Within the rectangular tile group, three green and three red tiles can be discarded as "0 pairs," leaving one green rectangular tile. The solution is $1n + 6$ as represented by one positive n-tile and six positive one tiles. This can also be written as $n + 6$.