11.2 teacher notes

Objectives

- Convert numbers between scientific notation and standard form.
- Multiply and divide numbers written in scientific notation.

Vocabulary

Standard form
Scientific notation
Power (Lesson 1-4)
Base (Lesson 1-4)
Exponent (Lesson 1-4)

Prerequisites

Applying rules of exponents

Understanding place value

Multiplying and dividing decimal numbers

Using the formula $d = rt$

Get Started

- Have students work in pairs.
- Write the following newspaper headline on the board or overhead projector:

  "World Population Hits 6.29 Billion"

- Ask students to work with their partners to write the number showing the world population. Tell them that this number—6,290,000,000—is written in standard form.
- Say, "There are many ways to express numbers besides in standard form. The form "6.29 billion" is one way to express a very large number. Can you see any advantages to the form "6.29 billion" over the form 6,290,000,000?" Guide students to see that the number 6.29 is not only easier to write but also easier for a newspaper reader to understand. Reading the word billion may have a greater impact on the reader than the zeros appearing in standard form.

- Say, "Today, we will learn another way to write numbers. It's called scientific notation. Scientific notation is often used to express numbers that are extremely small or extremely large. We will see how scientific notation can make calculations with these numbers easier."

Section 1

Expand Their Horizons

In Section 1, students will learn that scientific notation is a form of writing numbers that makes it easier to represent very large or very small values. A number is written in scientific notation when it is expressed as the product of a number *a* ($1 \leq a < 10$) and an integer power of ten. For example, 1.5×10^{-2} is scientific notation for the number 0.015. Additionally, 0.015 is called the standard form of the number. Scientific notation helps keep track of place value in numbers and is usually used to express numbers greater than one million or less than one millionth.

Students should use scientific calculators to explore scientific notation. Ask them to enter a multiplication problem such as $2{,}000{,}000 \times 9{,}000{,}000$ and to press the equal key. Notation on scientific calculators varies, but most will display "1.8 EE 13" or "1.8 E13." Point out that the product has too many digits to fit on the screen, so the calculator expresses the number in scientific notation. The notation "1.8 EE 13" or "1.8 E13" is equivalent to 1.8×10^{13} in scientific notation.

Additional Examples

1. **Discuss: Is 0.18×10^{-2} written in scientific notation?**

   No, because 0.18 is less than 1.

2. **Discuss: Would it be useful to use scientific notation when writing the number that expresses an average adult's weight in pounds?**

   No. It could be written in scientific notation (any number can), but it is not particularly useful because the weight of an average adult is not an extremely large or an extremely small number (typically considered to be numbers greater than one million or less than one millionth, respectively). It is easy enough to work with the weight value in standard notation.

Section 2

Expand Their Horizons

In Section 2, the concepts of multiplying and dividing numbers by powers of ten will be explored in order to help students learn to convert a number from scientific notation ($a \times 10^n$) to standard form. When converting to standard form, the decimal point in the number a is moved to the left or right n places. When n is positive, the decimal point is moved to the right; when n is negative, it is moved to the left. For example, to convert 9.3×10^7 to standard form, move the decimal point in 9.3 to the right seven places, adding zeros as needed. This yields $9.3 \times 10^7 =$ 93,000,000.

It is helpful for students to understand the arithmetic behind this method. First, remind students that writing the number 10^n in standard form (where n is a positive integer) yields the number one followed by n zeroes. For example, $10^7 = 10{,}000{,}000$. Therefore, the scientific notation expression 9.3×10^7 can be written as the product $9.3 \times 10{,}000{,}000$, which is equal to 93,000,000.

When converting a number written in scientific notation that has 10 raised to a negative power, the decimal point is moved to the left: 1×10^{-10} is written as 0.0000000001 in standard form. The value 10^{-10} also equals $\frac{1}{10{,}000{,}000{,}000}$. So, for example, $6.2 \times 10^{-3} = \frac{6.2}{1{,}000} = 6.2 \times 0.001 = 0.0062$.

Common Error Alert

Students may get confused when the left-hand factor of a number expressed in scientific notation does not have a decimal point. Remind them the decimal point, if it is not expressly written, is always located to the right of a number (to the right of the ones place).

Students will notice that positive exponents in scientific notation result in large numbers when written in standard form, and negative exponents in scientific notation result in small numbers when written in standard form.

Additional Examples

1. Write in standard form 2.11×10^0.

The exponent is 0; therefore, the decimal point is moved zero places. The resulting standard form is 2.11. This can be further verified because $10^0 = 1$, and $2.11 \times 1 =$ 2.11.

2. Write in standard form 7×10^8.

The exponent is positive; therefore, the decimal point is moved to the right eight places. Because a decimal point is not written with the seven, it should be placed to the right of the seven before moving the decimal point. The resulting standard form is 700,000,000.

Section 3

Expand Their Horizons

In Section 3, the methods learned for multiplying and dividing numbers by powers of ten will be used to convert a number from standard form to scientific notation. The first step in such a conversion requires the placement of a "new" decimal point to the right of the first nonzero digit in the standard form of the number (moving from left to right). This isolates the left-hand factor, *a*, for the scientific notation. Next, count from the new decimal point location to the number's original decimal point position. This step identifies the *n* part of the scientific notation, or the exponent of the 10. If the direction traveled from the new decimal point to the original is to the left, the power of ten is negative; if the direction traveled is to the right, the power of ten is positive.

For example in 0.0000002, place the new decimal point to the right of the two and then, count to the left seven places to the original decimal point. Moving to the left, seven places indicate that the scientific notation for 0.0000002 is 2×10^{-7}.

Another way to explain this procedure to students is to explain the arithmetic behind the method. The number 0.0000002 is two ten-millionths. In fraction form, two ten-millionths is or $\frac{2}{10{,}000{,}000}$ or $\frac{2}{10^7}$ or 2×10^{-7}.

1 **The number of meters in one micron is 1×10^{-6}. Write this number in standard form.** Though it is not shown, the number one implicitly has a decimal point to its right. The exponent of negative six indicates that the decimal point should be moved from that location to six places to the left, adding zeros as needed. The result is 0.000001. Remind students that a zero should always be placed in the ones position for proper form. The standard form for the length of a micron in meters is 0.000001 m. From the start, the negative exponent in the scientific notation of the number indicated that the standard form would be a small number.

2 **There are 110,000 hairs on an average human head. Write this number in scientific notation.** Place the "new" decimal point after the first nonzero digit. This would give 1.10000 or 1.1. Then, count over five decimal places to the right to get to the location of the original decimal point in 110,000. Moving five places to the right indicates that the exponent in the scientific notation is positive five, which yields 1.1×10^5.

Common Error Alert

Students sometimes have trouble correctly placing the new decimal point after the first nonzero number. Kinesthetic learners may find it helpful to draw a line through the number, from left to right, until their pencil arrives at the first number that is not zero. Have them draw a long vertical line to the right of that number, showing the location of the new decimal point.

Additional Examples

1. Write 0.00503 in scientific notation.

Looking from left to right, the first nonzero digit is five. The "new" decimal point is placed to the right of the five. To get from the new decimal point to the original, move to the left, three places. Therefore, the exponent of the 10 is negative three.

↓
0.00503
←←←

This is 5.03×10^{-3} in scientific notation.

2. Write 9.90 in scientific notation.

The decimal point is already correctly positioned to the right of the first nonzero digit. No move is necessary to get to the original point, so the exponent of the 10 is 0. This is 9.90×10^{0} in scientific notation.

Section 4

Expand Their Horizons

In Section 4, students will be shown how to multiply and divide numbers written in scientific notation. They will see that using scientific notation makes calculations with very large or very small numbers much easier to execute.

When calculating with scientific notation, it is helpful to recall that the product of $(a \times 10^m)$ and $(b \times 10^n)$ is $(ab) \times (10^{m+n})$. The quotient of $(a \times 10^m)$ and $(b \times 10^n)$ is $\left(\frac{a}{b}\right) \times (10^{m-n})$, where b is not equal to zero. The Associative Property of Multiplication allows the grouping of factors in this way, making it easier to multiply and to divide.

When two numbers in scientific notation are multiplied or divided together, the product or quotient itself is not necessarily a number in scientific notation. Referring to the rules above, it may not be true that $1 \leq ab < 10$ or $1 \leq \frac{a}{b} < 10$. If this is the case, the product or quotient should be adjusted so that it is written in scientific notation. Perform the adjustment by multiplying one factor by ten, and at the same time, by dividing the other factor by ten. The overall effect of these operations is to multiply the answer by one, so the value of the number is unchanged. For example, when calculating the quotient $\frac{1.5 \times 10^{14}}{3.0 \times 10^{8}}$, the result is 0.5×10^6, which is not in scientific notation. Multiplying 0.5 by 10 and dividing 10^6 by 10 does not change the value of the quotient but makes it possible to write it in scientific notation. The final answer is 5×10^5.

3 **Each side of a square microchip is 9×10^{-3} m long. Find the area of the microchip.** The product of (9×10^{-3}) and (9×10^{-3}) is 81×10^{-6}. While this product is correct, it is not expressed in scientific notation because 81 does not take the correct form $(1 \leq a < 10)$. Divide 81 by 10 to achieve the correct value. To compensate for dividing the first factor of the answer by ten, multiply the second factor by ten: $81 \times 10^{-6} = (81 \div 10) \times (10^{-6} \times 10) = 8.1 \times 10^{-5}$.

4 **The mass of Earth is 6×10^{27} g. The mass of the planet Pluto is 1.3×10^{25} g. How many times greater is Earth's mass than Pluto's mass?** Divide Earth's mass by Pluto's mass, using the rule $\frac{a \times 10^m}{b \times 10^n} = \left(\frac{a}{b}\right) \times (10^{m-n})$:
$\frac{(6 \times 10^{27})}{(1.3 \times 10^{25})} = \left(\frac{6}{1.3}\right) \times (10^{27-25}) \approx 4.6 \times 10^2$.
It is worthwhile to discuss with the class whether the quotient in this problem is better understood when it is expressed in scientific notation (Earth's mass is 4.6×10^2 times greater than Pluto's), or when it is expressed in standard form (Earth's mass is 460 times greater than Pluto's).

Additional Examples

1. Solve:

Mars is about 2.29×10^{11} m from the sun. If light travels at 3×10^{8} m/s, how long does it take sunlight to reach Mars?

Use the formula $t = \frac{d}{r}$:

$\frac{d}{r} = \frac{2.29 \times 10^{11}}{3 \times 10^{8}} = \left(\frac{2.29}{3}\right) \times (10^{11-8}) \approx$
$0.76 \times 10^{3} = 7.6 \times 10^{2}$

It takes sunlight about 7.6×10^{2} seconds to reach Mars.

2. Solve:

In one day, light travels about 1.61×10^{10} miles. How far will light travel in one year? (There are 365 days in a year.)

Before multiplying, write 365 in scientific notation:

$(1.61 \times 10^{10})(3.65 \times 10^{2}) =$
$(1.61 \times 3.65)(10^{10+2}) \approx 5.88 \times 10^{12}$

In one year, light travels about 5.88×10^{12} miles.

Connections

Many occupations use scientific notation to make it easier to write and calculate with very large or very small numbers. Astronomers use very large numbers when writing the number of stars in a galaxy or when calculating distances between planets. When dealing with weight capacity of steel beams or purchases of multi-million dollar properties, architects and real estate executives make use of scientific notation to represent large numbers. Microbiologists make use of scientific notation for calculations with very small numbers, such as the weight of a molecule.