11.1 teacher notes

Objectives

- Apply the *multiplication rule for exponents*.
- Apply the *division rule for exponents*.
- Apply the *definition of negative exponents*.
- Apply the *power-of-a-power rule*.
- Apply the *power-of-a-product rule*.
- Apply the *power-of-a-quotient rule*.

Vocabulary

Exponent (Lesson 1-4)
Power (Lesson 1-4)
Base (Lesson 1-4)
Multiplication rule for exponents
Division rule for exponents
Negative exponent
Power-of-a-power rule
Power-of-a-product rule
Power-of-a-quotient rule

Prerequisites

Adding, subtracting, multiplying, and dividing whole numbers

Get Started

- Write the expressions $(x \cdot x)(x \cdot x \cdot x)$, $\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$, and $(x \cdot x)(x \cdot x)(x \cdot x)$ on the board, while asking students to write each expression in exponential form.
- Student should obtain the following answers. The expression $(x \cdot x)(x \cdot x \cdot x) = x \cdot x \cdot x \cdot x \cdot x = x^5$; $\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = \frac{1 \cdot 1 \cdot x \cdot x \cdot x}{1 \cdot 1} = x \cdot x \cdot x = x^3$; and $(x \cdot x)(x \cdot x)(x \cdot x) = x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6$.
- For each problem, ask the class how the original problem could have been written using exponents. Show them $(x \cdot x) \cdot (x \cdot x \cdot x) = x^2 \cdot x^3$; $\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = \frac{x^5}{x^2}$; and $(x \cdot x)(x \cdot x)(x \cdot x) = x^2 \cdot x^2 \cdot x^2 = (x^2)^3$.
- Ask students to identify any correlation they see between the exponents in the original problem and the exponents in the answers. Promote discussion to help students recognize in the first problem five is the sum of two and three, in the second problem three is the difference of five and two, and in the third problem six is the product of two and three.
- In this lesson students will learn rules that simplify these problems in an efficient manner.

Section 1

Expand Their Horizons

In Section 1, students will learn to multiply powers with like bases. The term "like bases" means the factors raised to some power are identical. In the expression a^n, the value a is the base, and n is the exponent. For example, 14^8 and 14^6 have like bases, while 6^2 and 8^4 do not.

The *multiplication rule of exponents*, $a^m \cdot a^n = a^{m+n}$ $(a \neq 0)$, shows how to simplify an expression containing like bases. Recall a^m is a product where a is used as a factor m times, and a^n is a product where a is used as a factor n times. When a^m and a^n are multiplied together, a is used as a factor $m + n$ times, so the product is a^{m+n}.

The *division rule of exponents* states $\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$. There are m factors of a in the numerator and n factors of a in the denominator. In total there are $|m - n|$ factors. Absolute value is used because this number may be positive or negative. Here are some examples:

	Example
$m > n$	$\frac{2^4}{2^2} = 2^{4-2} = 2^2 = 4$
$m < n$	$\frac{2^2}{2^4} = 2^{2-4} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
$m = n$	$\frac{2^4}{2^4} = 2^{4-4} = 2^0 = 1$

Remind students that $a^0 = 1$ $(a \neq 0)$. Ask students *why* the expression 0^0 is undefined. Zero to any power is zero, and any number to the zero power is one. There is a contradiction then in the expression 0^0. Further, $0 = 1 - 1$; so, let 0^0 equal $(1 - 1)^{1-1}$. This equals $(1 - 1)^1(1 - 1)^{-1}$ or $\frac{1-1}{1-1}$, and this is undefined because any number divided by zero is undefined.

Negative exponents are defined in this lesson. Discuss the restriction placed on the variable in the rule $\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$. If $a = 0$, the expression is $\frac{0^m}{0^n}$ or $\frac{0}{0}$. Any value divided by zero is undefined. Students are sometimes confused by negative exponents, because they are not related to negative numbers. The value a^{-n} equals $\frac{1}{a^n}$. Again, this rule applies only when $a \neq 0$, because $\frac{1}{0^n} = \frac{1}{0}$ is not defined.

For base 10, $10^3 = 1{,}000$; $10^2 = 100$; and $10^1 = 10$. Continuing this process, $10^0 = 1$; $10^{-1} = 0.1 = \frac{1}{10}$; $10^{-2} = 0.01 = \frac{1}{100}$; and $10^{-3} = 0.001 = \frac{1}{1{,}000}$. Ask students to verify this on their calculator. They may recognize there are three factors of 10 in the numerator for 10^3, and there are three factors of 10 in the denominator for 10^{-3}. Similarly for 10^2, there are two factors of 10 in the numerator, and for 10^{-2}, there are two factors of 10 in the denominator. In general, a^{-n} equals $\frac{1}{a^n}$, where there are n factors of a in the denominator.

1 **Simplify: $4^3 \cdot 4$.** The factors have a like base of four; use the multiplication rule of exponents. Remind students four equals 4^1 not 4^0, because the value 4^0 equals one. Add the exponents. $4^3 \cdot 4^1 = 4^{3+1} = 4^4 = 256$.

2 **Simplify: $\frac{6^2}{6^5}$.** Since the terms in the numerator and denominator have like bases, use the division rule of exponents to simplify the expression. Subtract exponents and retain the common base. $\frac{6^2}{6^5} = 6^{-3}$. Then, using the rule of negative exponents, $6^{-3} = \frac{1}{6^3} = \frac{1}{216}$.

Additional Examples

1. Simplify: $2^4 \cdot 5^2$.

The factors 2^4 and 5^2 do not have the same base. The rules of exponents cannot be used to simplify the expression. However, $2^4 \cdot 5^2 = 16 \cdot 25 = 400$

2. Simplify: $\frac{4^{-3}}{2}$.

Notice, $4 = 2^2$; so, $4^{-3} = (2^2)^{-3} = 2^{-6}$. Now, apply the *definition of negative exponents*.
$\frac{2^{-6}}{2} = \frac{1}{2 \cdot 2^6} = \frac{1}{2^7} = \frac{1}{128}$

Section 2

Expand Their Horizons

In Section 2, students learn how to raise an exponential expression to a power and to simplify exponential expressions in which the base is a product or quotient. To raise an exponential expression to a power, use the rule $(a^m)^n = a^{mn}$. Emphasize to students this is derived from the fact $(a^m)^n$ is the same as $\underbrace{a^m \cdot a^m \cdot a^m \cdot \ldots \cdot a^m}_{n \text{ times}}$. Therefore, $(a^m)^n = a^{\overbrace{m+m+\ldots+m}^{n \text{ times}}} = a^{mn}$. For example, $(2^2)^3 = 2^2 \cdot 2^2 \cdot 2^2 = 2^{2+2+2} = 2^{2 \cdot 3} = 2^6$.

Students are familiar with the Distributive Property of Multiplication over Addition. The *power-of-a-product rule* is sometimes referred to as the Distributive Property of Exponentiation over Multiplication. To raise a product to a power, distribute the exponent to each factor. In the expression $(ab)^m$, distribute m to each factor a and b. The result is $a^m b^m$. Similarly, in the expression $\left(\frac{a}{b}\right)^m$, raise both the numerator and the denominator to the mth power to get $\frac{a^m}{b^m}$. This is true because $\left(\frac{a}{b}\right)^m = \left(a \cdot \frac{1}{b}\right)^m = \left(a \cdot b^{-1}\right)^m = a^m \cdot b^{-m} = \frac{a^m}{b^m}$. Notice, the latter rule does not apply when $b = 0$.

Connections

The rules of exponents are used when operations are performed on numbers written in scientific notation. To find the population density for China, divide the population by the land area. If China has a population of about 1.25×10^9 people and a land area of about 1×10^7 km^2, the population density is

$$\frac{1.25 \times 10^9}{1 \times 10^7} = \frac{1.25}{1} \cdot \frac{10^9}{10^7} = 1.25 \times 10^{9-7}$$
$$= 1.25 \times 10^2.$$

The population density of China, therefore, is about 125 persons per square kilometer.

3 **Simplify: $(4^3)^0$.** This problem shows an expression raised to the zero power. Any nonzero quantity raised to the zero power is one; $(4^3)^0 = 1$. The *power-of-a-product rule* can be applied to show the same result: $(4^3)^0 = 4^{3 \cdot 0} = 4^0 = 1$

4 **Simplify: $(3y^3)^2$.** In this problem a monomial is squared. When a monomial is raised to a power, each factor of the monomial is raised to that power. So $(3y^3)^2 = (3)^2 \cdot (y^3)^2$. Using the *multiplication rule for exponents*, $(3)^2 \cdot (y^3)^2 = 9 \cdot y^6 = 9y^6$.

5 **Simplify: $\left(\frac{4}{x}\right)^3$.** To simplify, use the *power-of-a-quotient rule*. Cube the numerator and cube the denominator, which gives $\left(\frac{4}{x}\right)^3 = \frac{4^3}{x^3} = \frac{64}{x^3}$.

Look Beyond

The rules of exponents are important in higher mathematics; they assist in the simplification of expressions and equations. As an example, to solve the equation $10^{x-1} = 100^{4-x}$, the equation can be written as $10^{x-1} = (10^2)^{4-x}$. Next, apply the *power-of-a-power rule* to get $10^{x-1} = 10^{8-2x}$. The left and right hand sides of the equation are equal, and the bases are equal. This means the exponents are equal as well, so $x - 1 = 8 - 2x$. This linear equation can be solved to find $x = 3$. Students at this point in algebra may not understand each step of this process, but they will generally understand the importance of these concepts in solving similar equations.

Additional Examples

1. Simplify: $(2^{-2})^3$.

Use the *power-of-a-power rule*. Then, apply the rule of negative exponents to simplify the base. Or, apply the *rule of negative exponents* to simplify the base

$2^{-2} = \frac{1}{4}$, and then cube the base.

$$(2^{-2})^3 = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$$

2. Simplify: $\left(\frac{2b}{5}\right)^2$.

Use the *power-of-a-quotient rule* to square the numerator and denominator. The numerator contains a monomial, so each factor must be squared.

$$\left(\frac{2b}{5}\right)^2 = \frac{(2b)^2}{5^2} = \frac{2^2b^2}{5^2} = \frac{4b^2}{25}$$