10.4 teacher notes

Objective

- Graph the solution set of a system of linear inequalities.

Vocabulary

Linear inequality (Lesson 7-3)
Boundary line (Lesson 7-3)
Slope-intercept form (Lesson 7-2)
Intercepts (Lesson 7-2)
System of linear inequalities

Prerequisites

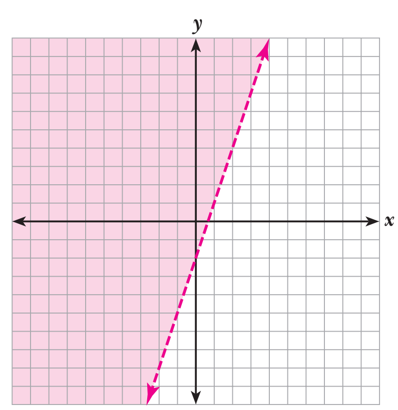
Graphing linear equations using the slope and y-intercept

Graphing linear equations using the intercepts

Graphing linear inequalities

Get Started

- Graph $y > 3x - 2$.
- Is the line solid or dashed? Explain. Dashed. $>$ and $<$ signs indicate dashed lines. $\geq$ and $\leq$ signs indicate solid lines.
- Which side of the graph is shaded? Explain. Shade the region that contains the point (0, 0). If the ordered pair (0, 0) is substituted in the inequality, the result is $0 > 3(0) - 2$, which is a true statement.



Section 1

Expand Their Horizons

In Section 1, students will be graphing systems of linear inequalities. It may be beneficial for the students to use colored pencils. Using a different color may make it easier to find the regions that overlap.

It is reasonable to expect some students to need a review of graphing linear equations. The first example is provided to help serve as a review. Pause the video when the line $y = \frac{2}{3}x - 3$ is graphed and allow students a chance to find the slope and the *y*-intercept for themselves.

The inequalities $y \geq -x + 4$ and $y > \frac{2}{3}x - 3$ are both graphed on the same coordinate plane. If students do not have colored pencils, then they should shade one of the inequalities with a pencil and shade the other inequality with a pen. If the students have only a pencil or a pen, the students should shade one of the inequalities using horizontal lines and the other inequality using vertical lines. The solution set will be the region that has overlapping colors or both vertical and horizontal lines.

For the second example, graph $y \leq -x + 6$. The boundary line has a slope of –1 and a *y*-intercept of 6. It is a solid line. The point (0, 0) can be used as the test point. $0 < -0 + 6$. $0 < 6$. Because this is a true statement, shade the region that contains the point (0, 0).

The boundary line of the second inequality, $y > \frac{1}{2}x$, has a slope of $\frac{1}{2}$ and a *y*-intercept of 0. The boundary line is a dashed line. The point (0, 0) cannot be used as the test point because (0, 0) lies on the line. Use a point not on the line, such as (0, 1). $1 > \frac{1}{2}(0)$. $1 > 0$. Because this is a true statement, shade the region that contains the point (0, 1). The region that has been shaded twice represents the solution set.

Common Error Alert

Many students may graph the boundary lines correctly but may shade the wrong region. It may be necessary to spend extra time explaining how to use the test points.

1 $y < x + 2$. The boundary line has a slope of 1 and a *y*-intercept of 2. The $<$ sign indicates that the line will be dashed. Pick a test point such as (0, 0). $0 < 0 + 2$. $0 < 2$. Because this is a true statement, shade the region that contains the point (0, 0). $y > -2x + 3$. The boundary line has a slope of –2 and a *y*-intercept of 3. The $>$ sign indicates that the line will be dashed. Pick a test point such as (0, 0). $0 > -2(0) + 3$. $0 > 3$. Because this is a false statement, shade the region that does not contain (0, 0). The solution set is the region that has been shaded twice.

2 $y \geq -3x + 4$. The boundary line has a slope of –3 and a *y*-intercept of 4. The $\geq$ sign indicates that the line will be solid. Pick a test point such as (0, 0). $0 \geq -3(0) + 4$. $0 \geq 4$. Since this is a false statement, shade the region that does not contain the point (0, 0). $y > x - 2$. The boundary line has a slope of 1 and a *y*-intercept of –2. The $>$ sign indicates that the line will be dashed. Pick a test point such as (0, 0). $0 > 0 - 2$. $0 > -2$. Because this is a true statement, shade the region that contains the point (0, 0). The solution set is the region that has been shaded twice.

Additional Examples

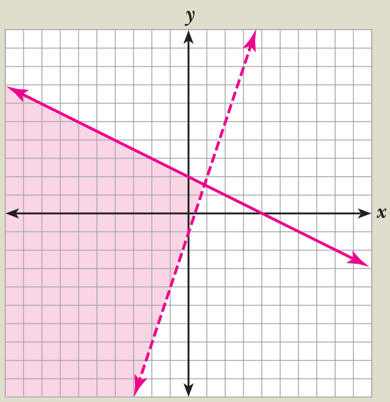
1. Solve by graphing:

$$\begin{cases} y > 3x - 1 \\ y \le -\frac{1}{2}x + 2 \end{cases}$$

$y > 3x - 1$
The boundary line has a slope of 3 and a y-intercept of –1. The boundary line is dashed. Pick a test point such as (0, 0). $0 > 3x - 1$. $0 > -1$. This is a true statement. Shade above the line.

$y \le -\frac{1}{2}x + 2$
The boundary line has a slope of $-\frac{1}{2}$ and a y-intercept of 2. The boundary line will be solid. Pick a test point such as (0, 0). $0 \le -\frac{1}{2}(0) + 2$. $0 \le 2$. This is a true statement. Shade below the line.

The solution set is the region that is shaded twice.



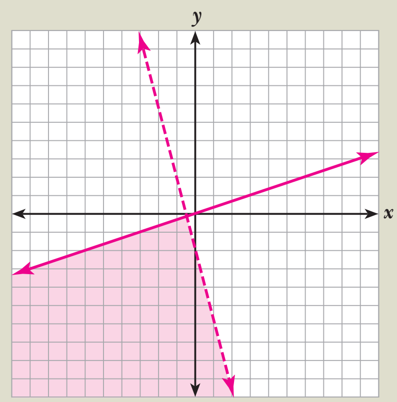
2. Solve by graphing:

$$\begin{cases} y \le \frac{1}{3}x \\ y < -4x - 2 \end{cases}$$

$y \le \frac{1}{3}x$
The boundary line has a slope of $\frac{1}{3}$ and a y-intercept of 0. The boundary line is solid. (0, 0) cannot be used as the test point. (0, 1) can be used instead. $1 \le \frac{1}{3}(0)$. $1 \le 0$. This is a false statement. Shade below the line.

$y < -4x - 2$
The boundary line has a slope of –4 and a y-intercept of –2. The boundary line is dashed. Use (0, 0) as the test point. $0 < -4(0) - 2$. $0 < -2$. This is a false statement. Shade below the line.

The solution set is the region that is shaded twice.



Section 2

Expand Their Horizons

In Section 2, students will graph systems of inequalities that include horizontal and vertical lines. Students will also graph systems of inequalities that have no solution.

Graph the boundary line $x = 2$. This is a vertical line. Remind students that points such as (2, 0), (2, 4), and (2, 6) lie on this line. If these points are connected, they form a vertical line. The boundary line will be solid. Use (0, 0) as a test point. Ask students, "Is 0 greater or equal to 2?" $0 \ge 2$ is false. Shade the region that does not contain the point (0, 0), that is, points to the right of the line $x - 2$.

Graph the boundary line $y = 3$. This will be a horizontal line. Points such as (–2, 3), (0, 3), and (2, 3) lie on this line. If these points are connected, they form a horizontal line. The boundary line will be solid. Use (0, 0) as a test point. Ask students, "Is 0 greater than or equal to 3?" $0 \geq 3$ is false. Shade the region that does not contain the point (0, 0)), that is, points above the line $y = 3$. The solution set is the region that is shaded twice.

Connections

To maximize hydroelectric power production, engineers use systems of inequalities in planning the location of dams.

The next system of inequalities has three inequalities. Students should use three colors to draw the graphs. The solution set is the region that has been shaded by all three colors. This region is a triangle.

The next example is a system of inequalities with parallel boundary lines. Students may assume that the solution set will be between the parallel lines. This is not necessarily the case. The solution set may also be above the top line, be below the bottom line, or be the empty set.

The test point (0, 0) in the first inequality produces $3(0) + 2(0) \leq 6$. $0 \leq 6$. The graph of this inequality is shaded below the boundary line.

The test point (0, 0) in the second inequality produces $3(0) + 2(0) \leq 12$. The graph of this inequality is also shaded below the boundary line. The solution set is the region that is shaded twice. This is the region below the inequality $3x + 2y \leq 12$.

The next example is also the graph of parallel lines. The first inequality is shaded below the boundary line. The second inequality is shaded above the boundary line. There is no region that is shaded twice. The solution set is the empty set.

3 The boundary lines of these inequalities form parallel lines. The first boundary line will be a solid line. The second boundary line will be a dashed line. The region that is shaded twice is the region between the lines.

4 This example also has boundary lines that are parallel. The shading is above the top line and below the bottom line. There is no solution.

5 Graph the boundary line at $x = 0$ and another line at $y = 0$. Both lines are dashed lines. The graphs of these lines lie on the axes. The region that is shaded twice is the entire third quadrant.

6 This system has four inequalities. The graph of the boundary lines will all be solid lines. There is a vertical line at $x = 1$, a vertical line at $x = 3$, a horizontal line at $y = 2$, and a horizontal line at $y = 5$. The region that is shaded four times is the rectangle bounded by the intersection of the lines.

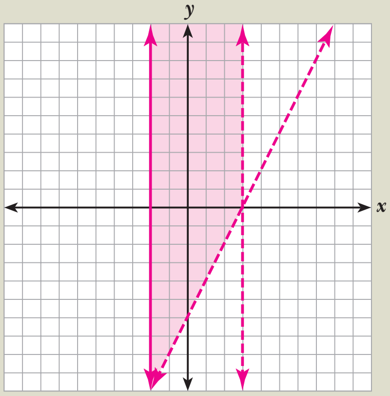
Look Beyond

In future mathematics courses, students will use systems of inequalities to solve linear programming problems. Linear programming is a method for solving problems that maximize or minimize a linear function (representing profit, cost, waste, mixture ingredients, etc.).

Additional Examples

1. Graph:

$$\begin{cases} x < 3 \\ x \geq -2 \\ 2x - y < 6 \end{cases}$$



2. Graph:

$$\begin{cases} 3x + y > 4 \\ y < -3x + 8 \end{cases}$$