10.3 teacher notes

Objective

- Solve systems of linear equations by substitution.

Vocabulary

Linear equation (Lesson 7-2)
Distributive Property (Lesson 2-3)
Inconsistent system (Lesson 10-1)

Prerequisites

Evaluating expressions

Solving linear equations of one variable

Solving an equation for a given variable

Get Started

- Explain how to find the value of the expression $4x - 3$ if $x = -2$? Substitute –2 for *x* and simplify. Try to have the students use the word "substitute."
- Find the value of the expression $4x - 3$ if $x = -2$. –11
- Simplify the expression $4x - 3$ if $x = y + 2$. $4(y + 2) - 3 = 4y + 5$

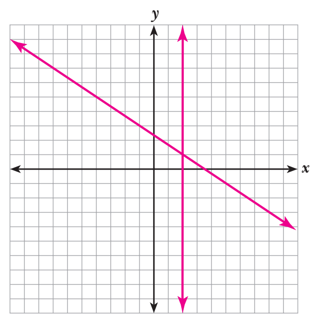
Section 1

Expand Their Horizons

In the previous lessons, students solved systems of equations by graphing and by elimination. Today, they will learn to solve systems of equations by substitution. Because different problems are solved more readily by one method or the other, it is necessary to learn all three methods.

Students often think that using a different method to solve a system of equations will produce a different result. It would be beneficial to work the first example using all three methods.

Graphing



Elimination

$$\begin{cases} 2x + 3y = 7 \\ x + 0y = 2 \end{cases}$$

$$-2(x + 0y) = -2(2)$$
$$-2x + 0y = -4$$
$$+\ 2x + 3y = 7$$
$$3y = 3$$
$$y = 1$$
$$x = 2$$

Substitution

$$\begin{cases} 2x + 3y = 7 \\ x = 2 \end{cases}$$

$$2(2) + 3y = 7$$
$$4 + 3y = 7$$
$$3y = 3$$
$$y = 1$$
$$x = 2$$

Note that each method produced the same result, (2, 1).

Common Error Alert

Students often forget to use the Distributive Property to evaluate expression such as $3(x - 2)$. Students may get $3x - 2$ instead of $3x - 6$. Remind them to use the Distributive Property. The parentheses mean that the three is multiplied by both x and -2.

Once again, it is important to check the solution in both equations.

In the second example, $y = x - 2$, it may be helpful to remind students of the composition of functions. They must substitute $x - 2$ in the second equation for y in the first equation, just as if they were finding $f(x - 2)$ for the variable y.

Substitute $x - 2$ for y in the first equation. $2x + 3(x - 2) = 9$. Simplify and solve to get $x = 3$.

Students need to find the value for y. Because the second equation is solved for y, it will be easier to use. However, the first equation will also yield the correct answer.

1 The value for y is given. Substitute 3 for y in the second equation. $3x - 2(3) = 6$. $3x - 6 = 6$. $3x = 12$. $x = 4$. It is given that $y = 3$. The solution is (4, 3). Check this solution.

2 Substitute $x - 3$ for y in the second equation. $x + (x - 3) = 5$. Simplify to get $x = 4$. Substitute this value for x in the first equation. $y = 4 - 3 = 1$. The solution is (4, 1). Check the solution.

Common Error Alert

Students may think they are finished after finding the value of only one variable. Remind them that the solution is an ordered pair.

Additional Examples

1. Solve by substitution:

$$\begin{cases} x = -5 \\ x + y = 12 \end{cases}$$

$$\begin{aligned} -5 + y &= 12 \\ y &= 17 \\ &(-5, 17) \end{aligned}$$

2. Solve by substitution:

$$\begin{cases} 2x - 3y = 8 \\ x = y - 3 \end{cases}$$

$$\begin{aligned} 2(y - 3) - 3y &= 8 \\ 2y - 6 - 3y &= 8 \\ -y - 6 &= 8 \\ -y &= 14 \\ y &= -14 \end{aligned}$$

$$\begin{aligned} x &= -14 - 3 \\ x &= -17 \\ &(-17, -14) \end{aligned}$$

Section 2

Expand Their Horizons

In Section 2, students will learn to use substitution to solve systems of equations that are not already solved for one of the variables. These problems are often in standard form and look like they could be solved by elimination. Assure them that using elimination would produce the same result, but if at least one of the variables has a coefficient of one, then it is sometimes easier to use substitution.

Common Error Alert

Students will sometimes solve an equation for a given variable and then try to substitute this expression into the same equation. In the first example, students substituted $4 - 2y$ for x in the second equation instead of the first equation. Be sure students use both equations.

In the first example, the second equation is $x + 2y = 4$, and because $2y$ is positive, it must be subtracted from both sides of the equation to get x by itself. $x + 2y = 4$. $x + 2y - 2y = 4 - 2y$. The result is $x = 4 - 2y$.

This result is then substituted for x in the first equation. $y = 1$. Substitute 1 for y in either equation. $x = 2$.

In the second example, it appears that it would be easier to solve for x in the first equation. However, this yields the equation $x = \frac{1}{2}y + 1$. Usually, most students prefer to avoid fractions whenever possible.

The second equation can easily be solved for y, $y = 2x - 2$. This expression is substituted into the first equation to find x. The result is $0 = 0$. Because this is a true statement, there are an infinite number of solutions to this system of equations. If these equations were graphed, the graphs would represent the same line. The solution can be written as $\{(x, y)\colon\ 6x = 3y + 6\}$ or $\{(x, y)\colon\ 2x - y = 2\}$.

The result of the substitution in the third example is $6 = 12$. Because this is a false statement, there is no solution to this system of equations. No solution can also be written as the empty set, $\{\}$ or $\varnothing$. The graphs of these equations are parallel lines.

In the fourth example, because both equations are solved for y, set the two expressions in x equal to each other. $3x - 13 = 2x - 8$. The solution is (5, 2).

3 The variable that is easiest to solve for is y in the second equation. This equation becomes $y = 2x - 1$. Substitute this expression into the first equation to get $3x + 4(2x - 1) = 18$. $x = 2$. Substitute $x = 2$ into either equation to find y. $y = 3$. Check the solution (2, 3) in both equations.

4 Multiplying the first equation by 2 results in the equation $6x + 2x = 4$. When the system is solved, the result is $4 = 7$. Because this is a false statement, the system has no solution.

Connections

Physicists use systems of equations in many situations. They use systems of equations to find mass, acceleration, velocity, force, and momentum.

For example, Newton's Second Law of Motion states that force equals mass times acceleration. If the same force were exerted on an object of mass 40g and an object of mass 50g with the acceleration of the first object being 30 m/s^2, then a system of equations could be used to find the acceleration of the second object.

$$F = 40(30)$$
$$F = 50a$$

$$40(30) = 50a$$
$$1200 = 50a$$
$$24 = a$$

The acceleration of the second object is 24m/s^2.

5 Both equations equal x. When $2y - 1$ is substituted for x in the first equation, the result is $2y - 1 = 3y + 7$. Solve for y. The solution is $y = -8$. When this solution is substituted into either equation, $x = -17$. The solution to the system is $(-17, -8)$. Check the solution in both equations.

Look Beyond

Students will learn different methods of solving systems of equations in future mathematics courses. Students will learn to use matrices (rectangular arrays of numbers arranged in rows and columns) to solve systems of equations.

The graphing calculator can be used to find the solution to systems of equations.

Eventually, students will solve systems of equations with more than two variables and two equations.

Additional Examples

1. Solve by substitution:

$$\begin{cases} x + 3y = 4 \\ -2x + 4y = 12 \end{cases}$$

$$x + 3y = 4$$
$$x = 4 - 3y$$

$$-2(4 - 3y) + 4y = 12$$
$$-8 + 6y + 4y = 12$$
$$-8 + 10y = 12$$
$$10y = 20$$
$$y = 2$$

$$x + 3(2) = 4$$
$$x + 6 = 4$$
$$x = -2$$
$$(-2, 2)$$

2. Solve by substitution:

$$\begin{cases} y = 3x + 4 \\ y = 3x - 6 \end{cases}$$

$$3x + 4 = 3x - 6$$
$$4 = -6$$

No solution