10.2 teacher notes

Objective

- Solve systems of equations using the elimination method.

Vocabulary

Variable (Lesson 2-1)
Coefficient (Lesson 2-1)
Ordered pair (Lesson 7-1)

Prerequisites

Evaluating expressions

Solving linear equations of one variable

Finding the least common multiple

Get Started

- Solve $2x = 6$. $x = 3$
- Solve $3x = 9$. $x = 3$
- Now add the two equations.

$$\begin{array}{r} 2x = 6 \\ (+)\ \ 3x = 9 \\ \hline 5x = 15 \end{array}$$

- Solve this new equation, $5x = 15$. What do you notice about all the solutions? $x = 3$. All the solutions are the same.

Section 1

Expand Their Horizons

In the first example, the equations $5x + 3y = 19$ and $-x - 3y = -11$ are added to eliminate the variable y. This addition results in an equation that contains one variable, x. That equation, $4x = 8$, is solved to get $x = 2$. However, this is only half of the solution. Remind students that a solution to a system of linear equations is an ordered pair. An ordered pair has both an x-coordinate and a y-coordinate. To find the value of y, substitute 2 for x in either equation. The solution is (2, 3). Have students follow along with the video and help them grasp the importance of checking solutions in each equation.

It will be beneficial to have students graph these equations to show that solving by graphing yields the same solution.

Common Error Alert

Students will multiply one side of the equation by a given number but not the other side. The Multiplication Property of Equality requires that both sides of the equation be multiplied by the same number.

In the second example, the equations are in two different forms. The first task is to rewrite the first equation in standard form so that both equations are in standard form. Because neither variable can be eliminated by just adding the equations, they must apply additional steps in this example. Help students see that they need to get either a pair of opposite x-coefficients or a pair of opposite y-coefficients. If there is a variable in the system with no coefficient appearing in front of it, then the coefficient is understood to be 1 or –1. Multiply that coefficient as needed to get a pair of opposite coefficients. In this example, multiply the first equation by 4.

Common Error Alert

Students will try to eliminate a variable by making both variables have exactly the same coefficient. By having them actually add the equations and write the $0x$ or the $0y$, they will remember that they must have opposites to obtain zero.

Substitute 4 for x in either original equation to find y.

1 In question one, the y terms are exactly the same. Either equation may be multiplied by –1. Then, y can be eliminated, and x is found to be 3. Substitute this value into either equation to find $y = \frac{1}{2}$. To check this solution, substitute these values into each equation. $3 - 8\left(\frac{1}{2}\right) = 3 - 4 = -1$. $4(3) - 8\left(\frac{1}{2}\right) = 12 - 4 = 8$. Both equations are true.

2 There is no need to multiply in the second example. The coefficients of the x's are opposites, and the x terms will be eliminated when the equations are added. $y = -5$. $x = -5$. The solution is (–5, –5). Check this solution by substituting –5 for x and y in each equation.

3 To eliminate the y's, multiply the first equation by 2 and then add the equations. Students may think that the solution (0, 0) is the same as no solution. If necessary, graph these equations to show that the graphs intersect at the origin. (0, 0) is the solution.

Common Error Alert

Students will often find the value of one variable but will neglect to solve for the other variable. Remind students that a solution to a system of equations is an ordered pair.

Additional Examples

1. Solve:

$$\begin{cases} \mathbf{x + 3y = 8} \\ \mathbf{-x - 2y = -5} \end{cases}$$

$$\begin{aligned} 0x + y &= 3 \\ y &= 3 \end{aligned}$$

$$\begin{aligned} x + 3(3) &= 8 \\ x + 9 &= 8 \\ x &= -1 \end{aligned}$$

The solution is (–1, 3).

Check:

$$\begin{aligned} x + 3y &= 8 \\ -1 + 3(3) &\stackrel{?}{=} 8 \\ 8 &= 8 \end{aligned}$$

True

$$\begin{aligned} -x - 2y &= -5 \\ -(-1) - 2(3) &\stackrel{?}{=} -5 \\ -5 &= -5 \end{aligned}$$

True

2. Solve:

$$\begin{cases} \mathbf{y = 2x + 3} \\ \mathbf{-6x + 2y = 4} \end{cases}$$

$$\begin{aligned} -2x + y &= 3 \\ -6x + 2y &= 4 \end{aligned}$$

$$\begin{aligned} -2(-2x + y) &= -2(3) \\ 4x - 2y &= -6 \\ \underline{-6x + 2y} &\underline{= 4} \\ -2x \quad &= -2 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} y &= 2(1) + 3 \\ y &= 5 \end{aligned}$$

The solution is (1, 5).

Check:

$$\begin{aligned} y &= 2x + 3 \\ 5 &\stackrel{?}{=} 2(1) + 3 \\ 5 &= 5 \end{aligned}$$

True

$$\begin{aligned} -6x + 2y &= 4 \\ -6(1) + 2(5) &\stackrel{?}{=} 4 \\ -6 + 10 &\stackrel{?}{=} 4 \\ 4 &= 4 \end{aligned}$$

True

Section 2

Expand Their Horizons

In Section 2, students will encounter systems of equations in which each equation must be multiplied by a different number to get a pair of opposite coefficients.

Students will learn to recognize that if the elimination method results in a false statement, then the system has no solution, and its graph is a pair of parallel lines. They will also learn to recognize that if the elimination method results in a true statement, then the system has an infinite number of solutions, and its graph is one line.

The first system of equations is solved in the video by eliminating the x terms. However, it could also have been solved by eliminating the y terms.

$$\begin{aligned} 4x + 3y &= 13 \\ -5x + 2y &= -22 \end{aligned} \qquad \begin{aligned} 2(4x + 3y) &= 2(13) \\ -3(-5x + 2y) &= -3(-22) \end{aligned}$$

$$\begin{aligned} 8x + 6y &= 26 \\ \underline{+\ (15x - 6y} &\underline{= 66)} \\ 23x \quad &= 92 \\ x &= 4 \end{aligned} \qquad \begin{aligned} 4x + 3y &= 13 \\ 4(4) + 3y &= 13 \\ y &= -1 \end{aligned}$$

Give students time to think about each step before proceeding to the next screen.

When the equations in the next example are added, the result is the false statement $0 = 5$. Because this statement is false, the system has no solution. From a common

Look Beyond

Students will use the elimination method learned in this unit to solve systems of more than two linear equations.

sense viewpoint, the first equation states that $6x + 4y = 5$, and the second equation states that $6x + 4y = 0$. $6x + 4y$ cannot equal both five and zero. Logically, this system cannot be solved. The graph of this system is a pair of parallel lines.

When the equations in the next example are added, the result is the true statement $0 = 0$. Because the statement is true, the system has an infinite number of solutions. Once again, logic can be used to find this solution. If the first equation is multiplied by 2, it is the same as the second equation. Any ordered pair that is a solution to one equation is a solution to the other equation.

4 In the video, to eliminate the y terms, Roxy multiplies the first equation by 7 and the second equation by 2. However, eliminating the x terms would have resulted in smaller numbers to work with. Either way is acceptable. Although it is not mentioned in the video, the solution should be checked.

Connections

A chemist can use a system of equations to find how much of one solution must be mixed with another solution to achieve a desired concentration.

5 If the first equation is multiplied by 2, it becomes the same as the second equation. This system of equations has an infinite number of solutions. The solution set may be written as $\{(x, y){:}\ x - y = 4\}$. If this system is graphed, the graph is a single line.

6 If the first equation is multiplied by 2, it becomes $2x + 4y = 6$. $2x + 4y$ cannot equal both 6 and 8. This system has no solution. If this system is graphed, the graph is a pair of parallel lines.

Additional Examples

1. Solve:

$$\begin{cases} \mathbf{3x + 5y = 6} \\ \mathbf{5x - 4y = 10} \end{cases}$$

$$4(3x + 5y) = 4(6)$$
$$5(5x - 4y) = 5(10)$$

$$\begin{array}{r} 12x + 20y = 24 \\ \underline{25x - 20y = 50} \\ 37x \qquad\quad = 74 \\ x = 2 \\ 3(2) + 5y = 6 \\ y = 0 \end{array}$$

(2, 0)

2. Solve:

$$\begin{cases} \mathbf{7y = 2x + 4} \\ \mathbf{-4x + 14y = 12} \end{cases}$$

$$-2x + 7y = 4$$
$$-4x + 14y = 12$$

$$-2(-2x + 7y) = -2(4)$$
$$-4x + 14y = 12$$

$$\begin{array}{r} 4x - 14y = -8 \\ \underline{-4x + 14y = 12} \\ 0 = 4 \end{array}$$

No solution