10.1 teacher notes

Objectives

- Determine whether a given ordered pair is a solution to a system of linear equations.
- Solve systems of two linear equations by graphing.
- Determine whether a system of linear equations is consistent, inconsistent, dependent, and/or independent.

Vocabulary

System of linear equations
Piecewise functions (Lesson 9-4)
Coordinate plane (Lesson 7-1)
Slope-intercept form (Lesson 7-2)
Consistent system
Inconsistent system
Dependent system
Independent system

Prerequisites

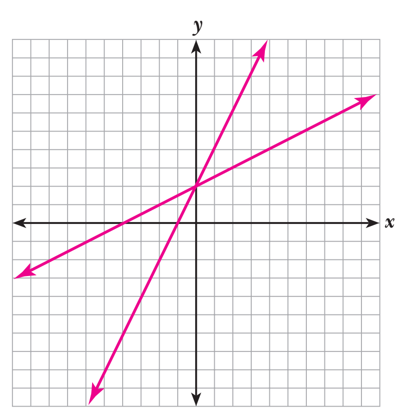
Evaluating expressions

Graphing linear equations using the slope and *y*-intercept

Graphing linear equations using the intercepts

Get Started

- Graph the line $y = 2x + 2$.
- Graph the line $y = \frac{1}{2}x + 2$ on the same coordinate plane.
- How many points of intersection do these lines have? one
- Is it possible to make two different lines intersect in two or more places? no
- Is it possible to have two lines in a coordinate plane that never intersect? yes, parallel lines



Section 1

Expand Their Horizons

The equations in a system are sometimes called simultaneous equations. The solution must satisfy all equations involved simultaneously. Graphing calculators can sometimes be used to solve systems of equations, too.

In Section 1, students will be given a system of two equations and a table of solutions for each equation. They will be asked to pick out the solution that appears in both tables and name it as the solution to the system. Then, students will be given an ordered pair and asked to determine if it is a solution to a given system of equations. Sometimes it is hard for students to grasp the concept that all the points on a line are solutions of the equation of the line. Checking these points by substituting their coordinates into the equation will reinforce the meaning of the term solution.

The systems of equations in this lesson are pairs of linear equations. The solution set to any system of equations consists of the point or points that satisfy both equations simultaneously.

In the first example, using the equations $y = -2x + 3$ and $y = -x + 1$, the characters in the video made a table to find the common solution. Students may become confused about how to find the numbers in the table. Help them to see that these numbers are only a few of the solutions to the equations. They can begin with any value for x and solve to find the corresponding value for y. The x-values in this example are chosen so that the solution to the system is among the ordered pairs.

The ordered pair (2, –1) is a solution to both equations. If the two equations were graphed, this would be the point of intersection of their lines.

For the next example, point out to students that the point (–1, –1) is substituted for the variables in both equations. Notice that, although this ordered pair satisfies the first equation, it does not satisfy the second equation.

1 Is (–4, 3) a solution to the system of linear equations? The coordinates make the first equation true and the second equation false. Therefore, (–4, 3) is not a solution for the system.

2 The ordered pair (2, 3) makes both equations true. Therefore, (2, 3) is a solution for the system.

Additional Examples

1. Is (2, –1) a solution to the system of linear equations?

$$\begin{cases} x + 2y = 0 \\ 3x - y = -7 \end{cases}$$

Check:

$$x + 2y = 0 \qquad 3x - y = -7$$
$$2 + 2(-1) \stackrel{?}{=} 0 \qquad 3(2) - (-1) \stackrel{?}{=} -7$$
$$0 = 0 \qquad 7 = -7$$

True   False

(2, –1) is not a solution.

2. Is (–3, 2) a solution to the system of equations?

$$\begin{cases} 2x + 4y = 2 \\ 4x - y = -14 \end{cases}$$

Check:

$$2x + 4y = 2 \qquad 4x - y = -14$$
$$2(-3) + 4(2) \stackrel{?}{=} 2 \qquad 4(-3) - 2 \stackrel{?}{=} -14$$
$$-6 + 8 \stackrel{?}{=} 2 \qquad -12 - 2 \stackrel{?}{=} -14$$
$$2 = 2 \qquad -14 = -14$$

True   True

(–3, 2) is a solution.

Section 2

Expand Their Horizons

In Section 2, students will learn to find the solution to systems of equations by graphing. The graph of an equation represents the solution set of the equation. If the graphs of two equations intersect, the point of intersection of the graphs represents the solution of the system.

Common Error Alert

Students often do not want to take the time to use a straight edge to draw the lines. Solving systems by graphing requires straight lines. Lines that are not straight may result in incorrect solutions.

The equations in the video are already in slope-intercept form. It may be necessary to review this form with students before proceeding. Remind students that slope is always written as a fraction. Therefore, the slope 2 in the first equation is written as $\frac{2}{1}$. This reminder may help students understand the movement of the point in the video.

In the first example, the video uses a blue line to represent the equation $y = 2x - 4$ and a green line to represent the equation $y = \frac{1}{2}x + 2$. Students will also benefit from using colored pencils for this lesson.

After finding the solution, it is very important to check the solution in each equation. Not only will the check ensure that the answer is correct, but it will also reinforce the students' understanding of what a solution to a system is.

There are three categories of systems of equations. If the graphs of the equation in a system intersect in exactly one point, then the system is said to be consistent and independent. If the graphs of the equations in a system coincide (meaning the graphs are really the same line), then the system is said to be consistent and dependent. If the graphs of the equations in a system are parallel lines, then the system is said to be inconsistent.

The system in the second example contains the equations $x + y = 5$ and $3x + 3y = 6$. These equations are not written in slope-intercept form. Therefore, it is easier for most students to graph this system using intercepts. Some students, however, may not be ready for the mental math that is used in the video. If not, it may be necessary to explain in more detail.

$$\begin{aligned} x + y &= 5 \\ 0 + y &= 5 \\ y &= 5 \\ &(0, 5) \end{aligned} \qquad \begin{aligned} x + y &= 5 \\ x + 0 &= 5 \\ x &= 5 \\ &(5, 0) \end{aligned}$$

$$\begin{aligned} 3x + 3y &= 6 \\ 3(0) + 3y &= 6 \\ 0 + 3y &= 6 \\ 3y &= 6 \\ y &= 2 \\ &(0, 2) \end{aligned} \qquad \begin{aligned} 3x + 3y &= 6 \\ 3x + 3(0) &= 6 \\ 3x &= 6 \\ x &= 2 \\ &(2, 0) \end{aligned}$$

These lines are parallel. This is an inconsistent system. Because it has no solution, it is neither dependent nor independent.

One way to indicate that this system has no solution is to write the symbol for empty set, { }. The empty set signifies that there is no solution. It contains no elements. The symbol $\varnothing$ can also be used to represent the empty set.

The next system, consisting of $4x + 6y = 12$ and $6x + 9y = 18$, is a dependent system. It has an infinite number of solutions. If these equations are written in slope-intercept form, they both become $y = -\frac{2}{3}x + 2$. This is a line with a y-intercept of 2 and a slope of $-\frac{2}{3}$. The solution set for this system consists of all ordered pairs whose points lie on this line. The solution set can be written in set notation: $\{(x, y): y = -\frac{2}{3}x + 2\}$. This set is read as, "the set of all x, y such that y equals negative two-thirds x plus two." Additional acceptable ways to write the solution set are $\{(x, y): 4x + 6y = 12\}$ and $\{(x, y): 6x + 9y = 18\}$. These equations are all equivalent.

Common Error Alert

Students will sometimes become confused by dependent systems of equations. They will assume that any system that does not have exactly one point of intersection has "no solution." Be sure to actually graph both equations. Students should see that the line contains an infinite number of solutions for each equation.

3 Some students may notice that the coefficients in the second equation are each three times the corresponding coefficients in the first equation. If the first equation is multiplied by 3, it becomes $6x + 3y = -6$. Except for the constant term, –6, this is the same as the second equation. Some students may intuitively discover that this system has no solution without graphing. This system is inconsistent. The solution set is the empty set, { }.

Look Beyond

Systems of equations can be solved using methods other than graphing. They can be solved by the algebraic methods of substitution and elimination, by using Cramer's Rule, by using augmented matrices, by using inverse matrices, and by using technology such as graphing calculators. Students will learn to use all of these methods in further studies of systems of equations.

4 This example has one equation in standard form and one equation in slope-intercept form. Graph the first equation using intercepts. The points with intercepts are (2, 0) and (0, –5). Graph the second equation using the slope and the *y*-intercept. The slope is $-\frac{3}{2}$ and the *y*-intercept is 3. These lines intersect at (2, 0). This solution is confirmed by substituting the coordinates into both equations. This system is consistent and independent.

5 This example also has one equation in standard form and one equation in slope-intercept form. These equations have the same line as their graphs. This is a dependent system. The solution set is $\{(x, y): y = 2x - 2\}$.

Connections

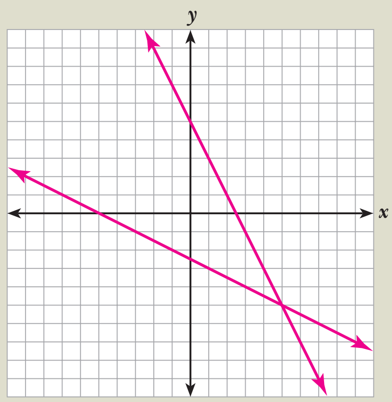
The graphs of systems of equations can be used by economists to model supply and demand. Supply and demand models are useful in forming business plans and budgets.

Systems of equations can also be used by dieticians to determine how many servings of certain foods are needed to ensure minimum nutrition requirements.

Additional Examples

1. Solve by graphing:

$$\begin{cases} 2x + y = 5 \\ x + 2y = -5 \end{cases}$$



These lines appear to intersect at (5, –5).

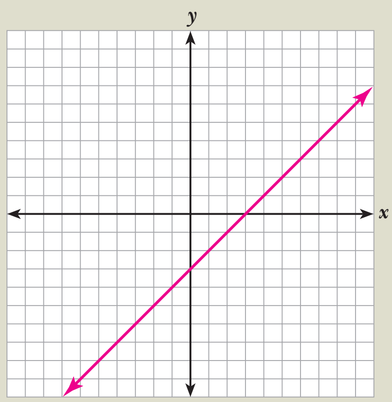
Check:

$$2x + y = 5 \qquad x + 2y = -5$$
$$2(5) + (-5) \stackrel{?}{=} 5 \qquad 5 + 2(-5) \stackrel{?}{=} -5$$
$$5 = 5 \qquad -5 = -5$$

The solution is (5, –5).

2. Solve by graphing:

$$\begin{cases} 4x - 4y = 12 \\ y = x - 3 \end{cases}$$



Both equations have the same graph. This system has an infinite number of solutions. It is consistent and dependent.