1.5 teacher notes

Objective

- **Apply the order of operations to evaluate mathematical expressions.**

Vocabulary

Exponent (Lesson 1-4)
Order of operations

Prerequisites

Simplifying expressions with integers

Simplifying expressions with rational numbers

Finding absolute value

Get Started

For which of these activities does order make a difference?

- Taking out the scraps and feeding the dog
- Brushing teeth and brushing hair
- Putting on shoes and putting on socks
- Doing Algebra homework and doing English homework
- Adding two numbers
- Multiplying two numbers
- Subtracting two numbers
- Dividing two numbers

Taking out the scraps and feeding the dog; putting on your shoes and putting on your socks; subtracting two numbers; dividing two numbers

Section 1

Expand Their Horizons

In Section 1, students will use the order of operations on integers without grouping symbols.

While solving the problem $7 + 2 \cdot 5$, it is possible that some students will get 45, instead of the correct answer 17. This discrepancy should show them the importance of learning the correct order of operations.

It may be fun for students to try to write their own mnemonic device for the order of operations, PEMDAS, instead of the one introduced in the lesson. This activity will also help students remember the correct order.

Common Error Alert

Using the acronym PEMDAS, students may try to add before they subtract and try to multiply before they divide instead of doing these pairs of operations from left to right. Stress that addition and subtraction have the same priority, and multiplication and division have the same priority.

Evaluate $2 - 10 \div 2 + 7$. At this point some students may still want to begin working this problem by subtracting 10 from 2. The correct order of operations is to do the division first. $2 - 5 + 7 = 4$.

Evaluate $20 - 10 + 19$. Do the operations from left to right. $10 + 19 = 29$.

1 $20 - 2 \cdot 2$. Do the multiplication first. $20 - 4 = 16$.

2 $16 \div -4 + 10(3)$. Work from left to right to do the division first and then the multiplication. Add last. $-4 + 30 = 26$.

3 $12 \div 4 \cdot 2(-1) \div 6$. Do all operations from left to right. $3 \cdot 2(-1) \div 6 = 6(-1) \div 6 = -6 \div 6 = -1$.

To evaluate $-7 + 3^2$, simplify the power first. $-7 + 9 = 2$.

To evaluate $-21 \div 7 + 2^3$, simplify the power first and then divide. $-21 \div 7 + 8 = -3 + 8 = 5$.

4 $12 - 4^2$. Simplify the power first, and then subtract. $12 - 16 = -4$.

5 $18 \div (-3)^2 - 7$. Simplify the power first, then divide and finally subtract. $18 \div 9 - 7 = 2 - 7 = -5$.

Additional Examples

1. Simplify:
$40 - 32 \div 2^3$

$40 - 32 \div 8 =$
$40 - 4 = 36$

2. Simplify:
$12 - (-2)^2 + 7$

$12 - 4 + 7 =$
$8 + 7 = 15$

Section 2

Expand Their Horizons

In Section 2, students will be using the order of operations to simplify expressions containing grouping symbols.

Another way to write the first example is $\frac{3+5}{2^2}$ or $(3+5) \div (2^2)$. $\frac{8}{4} = 2$.

Any time a fraction is used, it is understood that the numerator is grouped and the denominator is grouped.

Absolute value is also a grouping symbol. To simplify $8 \cdot 5 + |9 - 10|$, find the absolute value of $9 - 10$ first. $8 \cdot 5 + |-1| = 40 + 1 = 41$.

In the problem $(4)(5)^2$, the four and the five are not inside the same set of parentheses. Therefore, begin by finding five squared. $(4)(25) = 100$.

Common Error Alert

When given nested parentheses, students may not follow the order of operations correctly. Encourage them to identify the matching right parenthesis for each left parenthesis. This will help them identify the quantity to be evaluated.

6 $-1(1 + 5)^2 \div 9$. In this problem the parentheses are grouping symbols. Evaluate $1 + 5$ first. $-1(6)^2 \div 9 = -1(36) \div 9 = -36 \div 9 = -4$.

7 $\frac{9^2 - 3^2}{(-2)^4 + 11} = \frac{81 - 27}{16 + 11} = \frac{54}{27} = 2$.

Connections

Physicists, chemists, and biologists sometimes must use complicated formulas. In order to use the formulas correctly, these scientists must correctly apply the order of operations.

In the problem $288 \div [3(9 + 3)]$, one set of parentheses is nested inside the other. Perform the operation in the innermost set of parentheses first. Brackets are often used with nesting to avoid the confusion that two sets of parentheses might cause.

$288 \div [3(9 + 3)] = 288 \div [3(12)] = 288 \div 36 = 8$.

8 The $[3 - (-4)]$ is nested inside the problem. Do this operation first. $|9 \cdot 7| = |63| = 63$.

9 $[18 \div (9 - 7) + 6] - (4 - 10)$. Begin with the innermost set of parentheses, $(9 - 7)$. $[18 \div 2 + 6] - (-6) = [9 + 6] + 6 = 15 + 6 = 21$.

Look Beyond

In a later unit, students will use the Distributive Property. The Distributive Property provides an alternate way to evaluate some expressions. For example, $3(4 + 5) = 3(4) + 3(5) = 12 + 15 = 27$.

Additional Examples

1. Simplify: $\frac{12 + 4^2}{4 \cdot 7}$

$\frac{12 + 4^2}{4 \cdot 7}$

$\frac{12 + 16}{28}$

$\frac{28}{28} = 1$

2. Simplify: $|3 - 12| + 4^2 - [3 + (2 - 8)]^2$

$|3 - 12| + 4^2 - [3 + (2 - 8)]^2 =$

$|-9| + 16 - [3 + (-6)]^2 =$

$9 + 16 - (-3)^2 =$

$9 + 16 - 9 = 16$