1.4 teacher notes

Objectives

- Simplify expressions of the form b^n, where n is a whole number and b is a rational number.
- Simplify square roots and cube roots.

Vocabulary

Exponent
Base
Product (Lesson 1-2)
Square root
Radical sign
Radicand
Power

Prerequisites

Applying the Associative Property

Simplifying expressions with integers

Simplifying expressions with rational numbers

Get Started

- Find 2 · 2 · 2 · 2. 16
- Find (2 · 2) · (2 · 2). 16
- Find –2 · –2 · –2 · –2. 16
- Find (–2 · –2) · (–2 · –2). 16
- Why are all these answers the same? Answers may vary, but students should recognize the groupings and signs.

Section 1

Expand Their Horizons

In Section 1, students will simplify basic exponential expressions with positive numbers.

Common Error Alert

The most common mistake made by students when simplifying exponential expressions is multiplying the base and the exponent instead of using the base as a repeated factor. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$. $3^4 \neq 12$.

Use "squared" and "cubed" frequently to help stdents become familiar with the terms.

Five cubed is equal to five times five times five, or one hundred twenty-five.

The following sequence of steps may help students understand raising an expression to the zero power. Have students find the pattern.

$10^3 = 10 \cdot 10 \cdot 10 = 1{,}000$
$10^2 = 10 \cdot 10 = 100$
$10^1 = \quad = 10$
$10^0 = \quad = 1$

When the exponent is decreased by one, the final product is divided by ten.

1. $4^2 = 4 \cdot 4 = 16$
2. $8^0 = 1$. Any number raised to the zero power is equal to one.
3. $3^1 = 3$. Any number raised to the first power is equal to itself.
4. Another way to simplify $\left(\frac{1}{4}\right)^3$ is: $\left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3} = \frac{1}{64}$.

Additional Examples

1. Simplify:
$(4)^4$

$4^4 = 4 \cdot 4 \cdot 4 \cdot 4 = 256$

2. Simplify:
$(243)^0$

$(243)^0 = 1$

Section 2

Expand Their Horizons

In Section 2, students learn to simplify exponential expressions with bases that are negative.

Common Error Alert

Students using a calculator may type -2^4 to simplify $(-2)^4$. These are not equal expressions. -2^4 means $-(2)^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$.

It sometimes makes the computation easier to group the factors. Grouping factors in pairs will help students to discover that if a negative base is raised to an even power, the product will always be positive.

With the following expression, $(-2)^4 = -2 \cdot -2 \cdot -2 \cdot -2 = 16$, emphasize that parentheses are needed if the base is negative.

When negative two is raised to the fifth power, the product is negative.

$2^5 = (-2 \cdot -2) \cdot (-2 \cdot -2) \cdot -2 = -32$

When a negative number is raised to an odd power, the product is negative.

Encourage students to determine the sign of the product before the computation is performed. $(-4)^3$ will have a negative product because the exponent is odd. $(-4)^3 = -4 \cdot -4 \cdot -4 = -64$. $(-2)^6$ will have a positive product because the exponent is even. $(-2)^6 = (-2 \cdot -2) \cdot (-2 \cdot -2) \cdot (-2 \cdot -2) = 4 \cdot 4 \cdot 4 = 64$.

5 $(-1)^{14}$ is positive because the exponent, 14, is even.

6 $\left(-\frac{1}{3}\right)^3$ is negative because the exponent, 3, is odd. $\left(-\frac{1}{3}\right)^3 = -\frac{1}{3} \cdot -\frac{1}{3} \cdot -\frac{1}{3} = -\frac{1}{27}$.

Additional Examples

1. Determine the sign of $(-6)^3$. Then, simplify.

$(-6)^3$ is negative because the exponent is odd.
$(-6)^3 = -6 \cdot -6 \cdot -6 = -216$

2. Determine the sign of $\left(-\frac{2}{3}\right)^4$. Then, simplify.

$\left(-\frac{2}{3}\right)^4$ is positive because the exponent is even.
$\left(-\frac{2}{3}\right)^4 = \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) = \frac{16}{81}$

Section 3

Expand Their Horizons

In Section 3, students will find the square roots and cube roots of numbers.

The square root of 49 is 7. Using the square root sign indicates a nonnegative number called the principle square root. $\sqrt{49} = 7$. $-\sqrt{49} = -7$.

One way to find $\sqrt{49}$ is to ask, "What number, when multiplied by itself, has a product of 49?" This number is seven.

To find $\sqrt[3]{8}$, ask, "What number, when raised to the third power, is equal to eight?" $2 \cdot 2 \cdot 2 = 8$. Therefore, $\sqrt[3]{8} = 2$.

Positive numbers cannot be multiplied to get a negative product; therefore, $\sqrt[3]{-64}$ must be negative. $\sqrt[3]{-64} = -4$ because $-4 \cdot -4 \cdot -4 = -64$.

Look Beyond

Not all numbers are perfect squares or perfect cubes. Students will learn to simplify radical expressions that are not perfect squares or perfect cubes. For example, $\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6 \cdot \sqrt{2}$.

Connections

Engineers and scientists often work with numbers that are either too large or too small to write in standard form. They use engineering notation or scientific notation to represent these numbers. These systems are based on the powers of ten. For example, our sun is 93,000,000 miles from planet Earth. This distance can be represented in scientific notation as 9.3×10^7 miles. In engineering notation, it is 93×10^6 miles.

7 $\sqrt{100} = 10$ because $10 \cdot 10 = 100$.

8 $\sqrt[3]{27} = 3$ because $3 \cdot 3 \cdot 3 = 27$.

9 $\sqrt[3]{-216}$ will be negative. $\sqrt[3]{-216} = -6$ because $-6 \cdot -6 \cdot -6 = -216$. Students may find this problem more difficult than the others. Have students first guess the answer and check the guess. If the product is too low, guess a higher number. If the product is too high, guess a lower number until the correct answer is found.

Additional Examples

1. Simplify: $\sqrt{121}$

$11 \cdot 11 = 121$
$\sqrt{121} = 11$

2. Simplify: $\sqrt[3]{729}$

$9 \cdot 9 \cdot 9 = 729$
$\sqrt[3]{729} = 9$