$$\Delta = .00\,\pi + \frac{1}{200000\sqrt{xy}}$$

Objectives
- **Simplify expressions involving fractions.**
- **Simplify expressions involving decimals.**

$$\frac{1}{\Omega\,15750} \qquad \overline{5-6}\,|\,\sqrt{xy}\,\frac{1}{12}\,\triangle$$

1.3
teacher notes

Vocabulary
Least common denominator
Least common multiple
Factor

Prerequisites

Adding, subtracting, multiplying, and dividing integers

Finding the prime factorization of a whole number

Writing a whole number as a fraction with a given denominator

Reducing fractions to lowest terms

Finding the reciprocal of a number

Writing an improper fraction as a mixed number

Get Started

- Divide students in pairs.
- Have each student use a ruler to measure his or her pencil in inches.
- Have each pair add the lengths of the two pencils.
- Check the addition by measuring the pencils end to end.

Section 1

Expand Their Horizons

In Section 1, students will add, subtract, multiply, and divide signed fractions. This lesson includes a review of operations on fractions.

Some students struggle with fractions. Give the students plenty of time to work through the examples with the video.

Common Error Alert

When adding or subtracting fractions, students may forget to find a common denominator. Ask students to identify the operation before evaluating. If the operation identified is addition or subtraction, ask the students to check that the denominators are the same.

Although this section may be a review for some students, the additional practice will help them master the concepts behind simplifying numeric expressions involving fractions. This mastery will help them when they simplify algebraic rational expressions.

To find $\frac{1}{2}$ of $\frac{1}{4}$, multiply. $\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \left(\frac{1}{8}\right)$. To help students visualize this concept, actually draw the pizza and divide it into four equal pieces. Show them that $\frac{1}{2}$ of one of these pieces is $\frac{1}{8}$ of the pizza.

Add $\frac{1}{4}$ and $\frac{1}{8}$. A common denominator is needed to add fractions. Any common denominator will suffice, but the *least* common denominator is often the most convenient. The method shown in the video for finding the least common denominator is the same method used for denominators that contain variables. If students point out that the least common denominator for 4 and 8 is simply the greater of the two numbers, they

are correct *for this case.* However, the least common denominator is not simply the greatest of the denominators in all cases; this fact is illustrated just before the pause for the guided notes questions (with 12 and 15).

Find $1 - \frac{3}{8}$. To help students understand intuitively, ask, "Three-eighths of the pizza is gone. This is the same as three out of eight slices. How many slices are left?" Five slices are left. $\frac{5}{8}$ of the pizza is left.
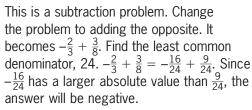
1 This problem is done by multiplying the numerator times the numerator and the denominator times the denominator. An alternative way is to cancel first. Five will divide into five one time and into 10 two times. Three will divide into three one time and into 6 two times.

$\frac{5}{6} \cdot \frac{3}{10} = \frac{1}{2}\frac{\cancel{5}}{\cancel{6}} \cdot \frac{\cancel{3}}{\cancel{10}}\frac{1}{2} = \frac{1}{4}$.

2 First find the least common denominator, 36. Multiply $\frac{5}{12}$ by $\frac{3}{3}$ to get $\frac{15}{36}$. Multiply $\frac{1}{18}$ times $\frac{2}{2}$ to get $\frac{2}{36}$. $\frac{15}{36} + \frac{2}{36} = \frac{17}{36}$. This fraction is already in simplest form.

3 This is a subtraction problem. Change the problem to adding the opposite. It becomes $-\frac{2}{3} + \frac{3}{8}$. Find the least common denominator, 24. $-\frac{2}{3} + \frac{3}{8} = -\frac{16}{24} + \frac{9}{24}$. Since $-\frac{16}{24}$ has a larger absolute value than $\frac{9}{24}$, the answer will be negative.

The next problem requires a subtraction and then a division. $1 - \frac{1}{4} = \frac{3}{4} \cdot \frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \cdot \frac{8}{1} = \frac{24}{4} = 6$. Remind students that division is the same as multiplying by the reciprocal. The reciprocal of $\frac{1}{8}$ is $\frac{8}{1}$.

4 $\frac{2}{3} \div \frac{2}{7} = \left(\frac{2}{3}\right)\left(\frac{7}{2}\right)$. If cancellation is performed first, the work is $\frac{2}{3} \cdot \frac{7}{2} = \frac{1}{3}\frac{\cancel{2}}{\cancel{2}} \cdot \frac{7}{1} = \frac{7}{3}$. $\frac{7}{3}$ can be changed to the mixed number, $2\frac{1}{3}$.

Additional Examples

1. Simplify:

$$-\frac{2}{3} + \frac{3}{4}$$

$$
\begin{array}{r}
-\frac{2}{3} = -\frac{8}{12} \\
+\frac{3}{4} = \frac{9}{12} \\
\hline
\frac{1}{12}
\end{array}
$$

2. Simplify:

$$\frac{3}{5} \div \left(-\frac{2}{7}\right)$$

$$\frac{3}{5} \div \left(-\frac{2}{7}\right) = \frac{3}{5} \cdot \left(-\frac{7}{2}\right) = -\frac{21}{10} = -2\frac{1}{10}$$

Section 2

Expand Their Horizons

In Section 2, students will be performing operations on signed decimals.

Common Error Alert

Students may confuse the placement of the decimal point in the answers for addition and multiplication of decimals. Decimal points must be lined up for addition. In multiplication the decimal places are counted.

Problems involving money are common decimal problems.

It is important when subtracting a decimal from a whole number to place zeros as place holders. For instance, $40 - 32.94$ would become $40.00 - 32.94$.

5 $0.3 + 0.03 + 23.456$. Begin by aligning the decimals. Although it is not necessary to use place holders in addition, it is a good habit for students to do so. The sum is 23.786.

6 $9.2 - 0.0054$. Placeholders must be used in this problem. Subtract $9.2000 - 0.0054$. The difference is 9.1946.

It is helpful to begin to use terms such as sum, difference, product, and quotient often so that students will become familiar with them.

Remind students that 15% is equal to 0.15. To change a percent to a decimal, move the decimal point two places to the left.

In the video problem, $(0.15)(\$77.94) = \$11.6910$, the product is rounded to $11.70. This may confuse some students. Generally, money is rounded up. This tip is arbitrary. In real life it makes more sense to round to a number that does not require pennies. Some people will give a $12.00 tip.

Divide 10.96 by 8. The decimal point is brought straight up from its original position.

To change a fraction into a decimal, divide the denominator into the numerator.

Be sure students move the decimal point before dividing by a decimal.

Common Error Alert

Students may write the quotient after the decimal point instead of placing the digits in their proper positions. In the first part of the example, 8 goes into 10 one time. The one should go directly above the zero in the 10. The decimal point is placed after the one.

For reinforcement, have students also divide $4\frac{1}{2}$ by $\frac{3}{8}$ using fractions.

$$4\frac{1}{2} \div \frac{3}{8} = \frac{9}{2} \div \frac{3}{8} = \left(\frac{9}{2}\right)\left(\frac{8}{3}\right) = \frac{72}{6} = 12.$$

7 $3.6 \times 0.25 = 0.900$. There are 3 decimal places in the product because the sum of the number of decimal places in the factors is 3.

8 Bring up the decimal point. 20 will not go into 3. Annex zeros after the 3 and put a zero in the quotient above the 3. The solution is 0.015.

Look Beyond

In a later unit, students will learn to add rational expressions such as $\frac{3}{x} + \frac{5}{2}$. They will add expressions like these using the concepts presented in this lesson. They will need to find a least common denominator and then add.

Students will also learn to subtract, multiply, and divide rational expressions that include numbers, variables, and exponents.

9 Begin by moving each decimal point three places to the right in order to divide by the whole number, 5. $400 \div 5 = 80$.

Connections

Stockbrokers and people who follow the stock market use operations on rational numbers daily. Stock values are given in dollars and cents. The change in a stock's value is given as a signed decimal — positive for an increase and negative for a decrease.

Meteorologists use operations on rational expressions to report and compare precipitation amounts. They also use rational expressions to determine average temperatures.

Additional Examples

1. Simplify:
$0.2 + 0.002 + 2$

$$
\begin{array}{r}
0.200 \\
0.002 \\
+\ 2.000 \\
\hline
2.202
\end{array}
$$

2. Simplify:
$(0.004)(3.4)$

$$
\begin{array}{r}
0.004 \\
\times\ \ \ \ 3.4 \\
\hline
16 \\
120 \\
\hline
0.0136
\end{array}
$$