1.2 teacher notes

Objectives

- Add two or more integers.
- Subtract integers.
- Multiply two or more integers.
- Divide integers.

Vocabulary

Integer (Lesson 1-1)
Absolute value
Addends
Product
Quotient
Sum
Difference

Prerequisites

Finding the absolute value of a number

Plotting integers on a number line

Get Started

- In a football game, what number would be used to describe a gain of 5 yards? 5
- What number would be used to describe a loss of 3 yards? –3
- How many total yards would be gained if a football team first gained 72 yards, then gained 5 more yards, and then lost 3 yards? 74 yards
- Represent this gain and loss of yards algebraically? $72 + 5 + (-3) = 74$

Section 1

Expand Their Horizons

In Section 1, students will be adding and subtracting integers. Students may have trouble with the concept that 500 − 100 = 400 can also be expressed as 500 + (–100) = 400.

Even though many students will be able to memorize the addition rules presented in this lesson, using a number line will help students visualize the concepts. By using a number line, students will better understand that subtracting an integer is the same as adding its opposite.

Students should recognize that a loss is represented by adding a negative number and a gain is represented by adding a positive number. After seeing the problem –5 + 2 = –3, many students will begin to realize that the sum of integers with different signs is found by subtracting the absolute values of the addends. The sum will take the sign of the addend with the greater absolute value.

On the number line, moving right represents adding a positive number and moving left represents adding a negative number.

The absolute value of a number is its distance from zero. Absolute value is always nonnegative. The absolute value of 5 is 5. This is written as $|5| = 5$. The absolute value of –5 is also 5. This is written as $|-5| = 5$. The absolute value of 0 is 0. This is written $|0| = 0$.

To add –6 + (–2) + 1, begin by adding from left to right. Add the first two numbers to get –6 + –2 = –8. Then, add this sum –8 to the last number. To find the sum of –8 and 1, subtract their absolute values. The sum will be negative because –8 has a larger absolute value than 1. 8 − 1 = 7, –8 + 1 = –7

1. 10 + (–3). The signs are different. Subtract their absolute values to find the answer. 10 – 3 = 7. Ten has the larger absolute value. The sum is positive seven.
2. –42 + (–8). The signs are the same. Add absolute values to find the answer. 42 + 8 = 50. Both addends are negative. The sum is –50.
3. –15 + 7. The signs are different. Subtract their absolute values to find the answer. 15 − 7 = 8. Negative fifteen has the larger absolute value. The sum is –8.
4. 12 + (–6) + 1 + (–7). Begin by adding the first two integers, 12 + (–6) = 6. Now add, 6 + 1 = 7. Then, add 7 + (–7). The sum is zero.

Subtracting a number is the same as adding its opposite. Therefore, 10 − 3 = 10 + (–3) = 7.

5 − 4 ≠ 4 − 5. Subtraction is not commutative. If the temperature is 5° and then drops by 4°, the resulting temperature is 1°. But if the temperature is 4° and then drops by 5°, the resulting temperature is –1°.

Common Error Alert

For an expression like –3 − 9, students may subtract absolute values to get the wrong anwer of –6 or 6. Encourage students to rewrite the expression as –3 + (–9) and add absolute values to get the correct answer of –12.

In an expression such as 18 − (–10) = 18 + 10 = 28, students will often wonder why the difference is larger than the original number. Subtracting negative ten is the same as adding positive ten.

5. –3 − 9 = –3 + (–9) = –12. Subtracting 9 is the same as adding –9.
6. 25 − 40 = 25 + (–40) = –15. For emphasis, explain to students that if the temperature is 25° and then drops by 40°, the resulting temperature is –15°.
7. 12 − (–12) = 12 + 12 = 24. Subtracting –12 is the same as adding 12.
8. –6 − (–18) = –6 + 18 = 12. First add the opposite of –18 which is 18, and then add integers with different signs to get 12.

Additional Examples

1. Simplify:
–2 + 9

The signs are different. Subtract absolute values.
9 – 2 = 7
9 has a larger absolute value than –2.
The answer is positive 7.
–2 + 9 = 7

2. Simplify:
–6 – (–2)

–6 – (–2) = –6 + 2 = –4

Section 2

Expand Their Horizons

In Section 2, students will multiply and divide integers. The rules for multiplying two numbers and for dividing two numbers are the same.

1) Perform the indicated operation with absolute values.
2) If the signs are the same, the answer is positive.
3) If the signs are different, the answer is negative.

Common Error Alert

After using parentheses to indicate multiplication, students will often mistakenly multiply when given a problem such as (–2) – (–3). Students should identify which operations are in an expression before evaluating the expression.
(–2) – (–3) = –2 + 3 = 1.

The first two examples in the video are explained by repeated addition. The third example is explained by showing a pattern. All three examples are consistent with the rules.

In problems with three or more factors, the product of the first two factors should be found. This product is then multiplied by the third factor, and so on until all factors have been multiplied.

If there is an odd number of negative factors, the product will be negative. If there is an even number of negative factors, the product will be positive.

One way to find the product (–1)(5)(–3) is to multiply absolute values. (1)(5)(3) = 15. Then count the negative signs, 2. The number of negative factors is even, so the product is positive. (–1)(5)(–3) = 15.

9 (–9)(11) = –99. The signs are different. Therefore, the product is negative.

10 (–4)(–2) = 8. The signs are the same. Therefore, the product is positive.

11 To find the product (–10)(–6)(–2), first find (–10)(–6). (–10)(–6) = 60. Complete the multiplication. 60(–2) = –120.

12 To find the product (5)(0)(–18), first find (5)(0). (5)(0) = 0. Complete the multiplication. 0(–18) = 0. Anytime one of the factors of a multiplication problem is zero, the product is zero.

Division by a number is the same as multiplying by its reciprocal. $4 \div (-2) = 4 \cdot \frac{-1}{2} = -2$. However, unless fractions are involved, it is usually easier to simply divide.

In the expression 8 ÷ (–4) = –2, the signs are different. Therefore, the quotient is negative.

Connections

People who keep the statistics at a football game use integers to describe the total number of yards gained and lost by a particular team or team member. It is much simpler for them to denote a five yard gain as a positive five and a five yard loss as a negative five instead of trying to keep up with a gain and lost column. At the end of the game or the end of the season, the figures can be tallied to find the net yardage.

In the expression, $\frac{0}{-3} = 0$. Zero divided by any nonzero number is zero.

A number cannot be divided by zero. Therefore, $\frac{a}{0}$ is undefined.

13 $\frac{-25}{-5} = 5$. The signs are the same. The quotient is positive.

14 $-81 \div 9 = -9$. The signs are different. The quotient is negative.

15 $\frac{160}{0}$. A number cannot be divided by zero. The quotient is undefined.

Look Beyond

In a later unit, students will be performing operations on expressions that include variables and exponents. They will be simplifying expressions such as $3x^2(-2x)$. $3x^2(-2x) = -6x^3$. Learning multiplication and division rules in this lesson will make simplifying these more complex expressions easier.

Additional Examples

1. Simplify:
(–4)(12)

$(-4)(12) = -48$
The signs are different. The product is negative.

2. Simplify:
(–125) ÷ (–5)

$(-125) \div (-5) = 25$
The signs are the same. The quotient is positive.