1.1 teacher notes

Objectives

- Use appropriate set notation.
- Use Venn diagrams to show set relationships.
- Describe sets of numbers.
- Classify numbers into sets.
- Graph numbers on a number line.

Vocabulary

Set
Subset
Element
Disjoint sets
Venn diagram
Intersection
Empty set or Null set
Union
Natural number or Counting number
Whole number
Integer
Rational number
Irrational number
Real number
Number line

Prerequisites

Converting a decimal to a fraction

Finding the rational square root of a number

Get Started

- Sets are all around us. Name some things that come in sets. Dishes, silverware, tools, tires, etc.
- Name some elements in the set of equipment necessary for a baseball game. Baseball, bat, glove, bases, etc.

Section 1

Expand Their Horizons

In Section 1, students will learn some basic set theory. They will begin with sets of familiar objects. Students should be encouraged to think of other groups of objects that are sets.

Commas separate the elements of a set. The curly brackets "{ }" are used to show sets. The order in which the elements are given does not matter. For example, the sets {3, 5} and {5, 3} are the same.

Any member of a set is an element of that set. The Greek letter epsilon, "∈", means "is an element of." To draw an epsilon, make a capital "C" and put a line in the middle of it.

A subset of a set B can be formed with any of the elements in set B. Every set is also a subset of itself. The empty set is a subset of all sets.

1 Any of the four elements of set X is a correct answer. Encourage students to use the "∈" symbol to write their answers.

2 There are many correct answers. The subset L given on the video is a sample. Acceptable answers include {1}, {3, 5}, and {1, 3, 5, 7}. Students should use brackets around the subset.

In the next section, students are introduced to Venn diagrams. Write "Venn diagram" on the chalkboard. Sets in a Venn diagram are represented by circles placed inside a rectangle. Elements in each set are contained within their respective circles. If two sets share one or more common elements, the circles intersect. The intersection of the circles represents the set of common elements. The set of common elements is called the intersection of the two sets. If the sets are disjoint, the circles will not intersect.

3 Most students will quickly recognize that A and X are not disjoint sets. However, it is important that the students explain their reasoning. Students should use complete sentences in the explanation.

When making a Venn diagram, students should first identify the common elements. These elements should be written in the area where the two circles intersect. The non-common elements of the sets should be written within their respective parts of the circles that do not intersect. The common elements should be written only one time. These ideas are illustrated on the video in the Venn diagram for sets A and X.

Common Error Alert

Students often confuse intersection and union. When people join a union, they unite for a common cause. When there is a union of sets, the elements in the sets are combined or united. Remember the letter "U" for union.

When two sets have no elements in common, their intersection is the empty set, or null set. The empty set can be written as either { } or ∅.

When students combine sets to form a union, they will often write common elements twice. Having them write the elements in increasing order will help guard against writing an element more than once.

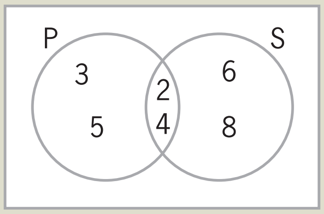
Additional Examples

1. Name a subset of R.
R = {3, 6, 9, 12, 15, 18}

Accept all answers that are subsets of R. Possible answers are: {6}, {9, 12}, {6, 15, 18}, {3, 9, 12, 15, 18}

2. Draw a Venn diagram for sets P and S. Name the intersection.
P = {2, 3, 4, 5}
S = {2, 4, 6, 8}

$P \cap S = \{2, 4\}$



Section 2

Expand Their Horizons

In Section 2, students will work with infinite sets. The infinite sets in Section 2 are used to classify numbers.

The natural numbers set is the set of numbers used to count objects.

Natural numbers are indicated by {1, 2, 3, 4, …}. The dots indicate that the set is infinite and the list of elements in the set follows the pattern established by 1, 2, 3, 4.

When the number zero is added to the set of natural numbers, the set of whole numbers is formed.

Students at this level should be familiar with negative numbers. When the opposites of the counting numbers are added to the set of whole numbers, the set of integers is formed.

Students may question the use of the letter Z to stand for integers. Both integers and irrational numbers start with the letter I. The letter Z is often used for integers. There is no commonly used letter for the set of irrational numbers. The letter S was chosen for the video.

4 Some additional answers are losing yards in a football game, losing weight, and withdrawing more money than was in a bank account.

Additional Examples

1. The numbers on a scoreboard at a basketball game are best represented as elements of which real number set? Explain your answer.

Since there are no negative numbers on a scoreboard and there cannot be a fractional part of a point, the whole numbers are the best fit.

2. Give an example of a number that is a natural number, a whole number, and an integer.

Any natural number is a correct response.

Section 3

Expand Their Horizons

In Section 3, students will learn about rational numbers and irrational numbers. These sets may be more abstract to the students.

Any number that can be written as a fraction with an integer numerator and nonzero integer denominator is a rational number. Decimals that either terminate or repeat are rational numbers. Natural numbers, whole numbers, and integers are all rational numbers.

The following statement, $\{\frac{a}{b}: a, b \in \mathrm{Z}; b \neq 0\}$, is read as *the set of all expressions* a *divided by* b *such that* a *and* b *are elements of the set of integers and* b *does not equal zero.* Make sure students understand why b is nonzero. For example, if $\frac{5}{0} = n$ then $5 = 0 \cdot n$, but no number n multiplied by zero would yield another product other than zero.

5 Some additional answers are on a fever thermometer, in baseball statistics, and in cooking measurements.

The Pythagoreans are best known for the Pythagorean Theorem. The Pythagorean Theorem states that in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the hypotenuse. This theorem is usually written as $a^2 + b^2 = c^2$. Using this theorem may produce an irrational number. For more information on the Pythagoreans, see the Look Beyond section of the Teacher Notes.

The square root of a whole number is rational if the whole number is a perfect square. For example, $\sqrt{9}$ is rational because $9 = 3^2$. The square root of a whole number is irrational if the whole number is not a perfect square. For example, $\sqrt{7}$ is irrational. Another irrational number students may be familiar with is π.

The sets of rational numbers and irrational numbers are disjoint sets. The union of the sets of rational numbers and irrational numbers is the set of real numbers.

Students may ask if there are numbers that are not real. There is a set of such numbers. It is called the set of imaginary numbers. The set of imaginary numbers is based on the number i. The number i is the square root of negative one.

Additional Examples

1. To which real number sets does –0.5 belong?

$-0.5 \in \mathrm{Q}$
$-0.5 \in \mathrm{R}$

2. To which real number sets does $\sqrt{36}$ belong?

$\sqrt{36} \in \mathrm{N}$
$\sqrt{36} \in \mathrm{W}$
$\sqrt{36} \in \mathrm{Z}$
$\sqrt{36} \in \mathrm{Q}$
$\sqrt{36} \in \mathrm{R}$

Section 4

Expand Their Horizons

Any real number can be graphed on a real number line. However, estimation is necessary for numbers that are not integers.

Some students may have difficulty graphing $\sqrt{8}$. The numbers closest to 8 that have integer square roots are 4 and 9. $\sqrt{4} = 2$. $\sqrt{9} = 3$. Therefore, $\sqrt{8}$ is between 2 and 3. Since 8 is closer to 9 than to 4, $\sqrt{8}$ is closer to 3 than to 2.

Look Beyond

The Pythagoreans were the followers of Pythagoras, a Greek philosopher and mathematician, who lived from approximately 572–500 BC. Pythagoras formed an academy in order to study philosophy, mathematics, music, and astronomy. The Pythagoreans believed that through numbers humanity and matter could be understood. They lived in secrecy and all teaching was oral; they did not write down their findings for fear that they would be found. Despite their eccentric tendencies, Pythagoras and his followers have left us with many useful discoveries. Most notably, they discovered the Pythagorean Theorem.

Common Error Alert

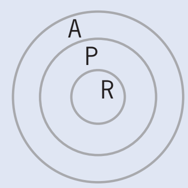
Students may move in the wrong direction on the negative portion of the number line. For example, they may place –4.2 closer to –5 than to –4.

Connections

A Venn diagram can be used to test the validity of a logical argument. For example:

All parallelograms have two pairs of congruent sides. All rectangles are parallelograms. Therefore, all rectangles have two pairs of congruent sides. The Venn diagram shows that the argument is valid, because R is a subset of A.

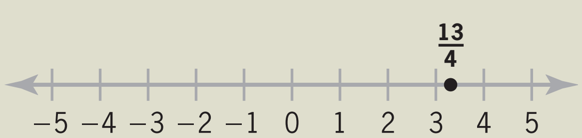
A = the set of all figures with two pairs of congruent sides
P = the set of all parallelograms
R = the set of all rectangles



Additional Examples

1. Graph $\frac{13}{4}$ on the number line.

$\frac{13}{4} = 3\frac{1}{4}$



2. Graph $\sqrt{13}$ on the number line.

The numbers closest to 13 that have integer square roots are 9 and 16.

$\sqrt{9} = 3$
$\sqrt{16} = 4$

13 is a little closer to 16 than to 9. $\sqrt{13}$ is a little closer to 4 than to 3.